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ABSTRACT

This concept module on force is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. This module discusses the concept of force and forcelike quantities as they apply to four energy systems: mechanical, fluidal, electrical, and thermal. The forcelike quantities as discussed are force, torque, pressure, voltage, and temperature difference. Included with the concept module are eighteen application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Gravity: Addition of Mechanical Forces: Addition of Torques: Mechanical Equilibrium in Structures: Stress and Strain: Tension in Wires and Cables: Atmospheric Pressure: Density, Buoyancy, and Specific Gravity: Hydraulic Jack: Manometers: Potential Difference and Charge Flow: Series and Parallel Voltage Sources: Electric Forces and Fields: Magnetic Forces and Fields: Lead-Acid Storage Battery: Thermometers: Temperature Differences and Heat Flow: and Thermal Stress. (LRA)

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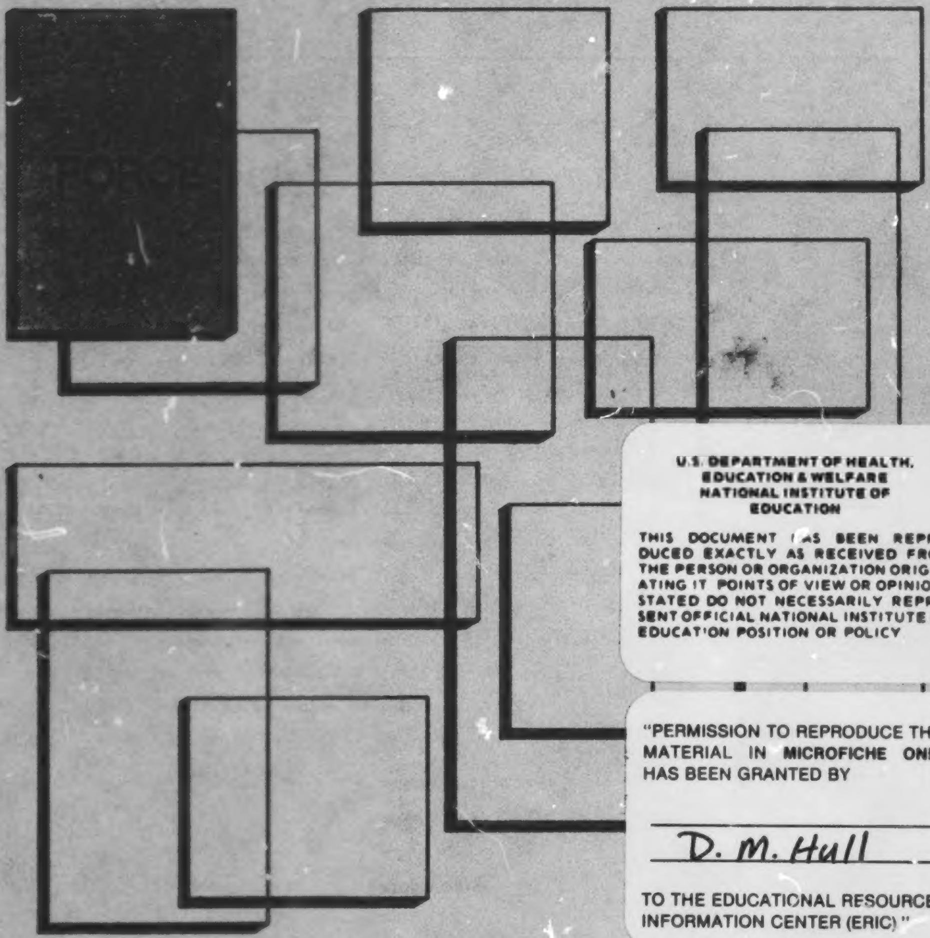
UNIFIED TECHNICAL CONCEPTS

CONCEPT MODULE 1 - 0



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INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet specific needs of students in a specific technology.

STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems force and torque cause motion of mass.
- In fluidal systems pressure difference causes motion of fluids.
- In electrical systems potential difference causes motion of mass.
- In thermal systems temperature difference causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

COMPONENTS OF UTC

UTC incorporates three types of modules.

CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires In the laboratory, students calculate and measure experimentally the resistance of various wires.

SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

INTRODUCTION

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

PREREQUISITES

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

OBJECTIVES

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

DISCUSSION

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

LABORATORY (called DEMONSTRATIONS in the Concept Modules)

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

SELF CHECK

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

REFERENCES

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS

1-0 Force
1M1 Gravity
1M2 Addition of Mechanical Forces
1M3 Addition of Torques
1M4 Mechanical Equilibrium in Structures
1M5 Stress and Strain
1M6 Tension in Wires and Cables
1F1 Atmospheric Pressure
1F2 Density, Buoyancy, and Specific Gravity
1F3 Hydraulic Jack
1F4 Manometers
1E1 Potential Difference and Charge Flow
1E2 Series and Parallel Voltage Sources
1E3 Electric Forces and Fields
1E4 Magnetic Forces and Fields
1E5 Lead-Acid Storage Battery
1T1 Thermometers
1T2 Temperature Difference and Heat Flow
1T3 Thermal Stress

INTRODUCTION

The concept of force is based upon the common experiences of every person. A force is a push or pull that may cause an object to move. Force — a physical quantity that causes movement — is the "prime mover" in the mechanical energy system.

This module discusses the concept of force and forcelike quantities as they apply to four energy systems — mechanical, fluidal, electrical and thermal. The forcelike quantities discussed are force, torque, pressure, voltage and temperature difference.

- Force causes motion of an object in a translational mechanical system. This force serves as the model for all forcelike quantities.
- Torque causes rotation of an object in a rotational mechanical system. Torque is a forcelike quantity.
- Pressure causes a fluid to move in a fluidal system. Pressure is a forcelike quantity.
- Voltage causes electrical charge to move in an electrical system. Voltage is a forcelike quantity.
- Temperature difference causes heat energy to move in a thermal system. Temperature difference is a forcelike quantity.

The Discussion includes the mathematical formulation and units used to describe each forcelike quantity in each energy system and stresses the characteristics of forcelike quantities that apply to all energy systems.

PREREQUISITES

One year of high school algebra is recommended.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following physical quantities and, where applicable, state their units in both SI (International System of Units) and English System of Units.

  Force.
  Torque.
  Pressure.
  Voltage.
  Temperature difference.

- Given two or more mechanical forces acting along the same line, determine the resultant force.

- Given two of the following quantities in a mecanical rotational system, determine the third:

  Force.
  Lever arm.
  Torque.

- Given two of the following quantities in a fluid system, determine the third:

  Force.
  Area.
  Pressure.

- Given two of the following quantities in a fluid system, determine the third:

  Pressure
  Height of fluid.
  Weight density.

- Given two or more voltage sources connected in series, determine the resultant voltage.

- Given a temperature in either degrees Celsius or degrees Fahrenheit, determine the equivalent temperature on the other scale.

- Describe how pressure in fluidal systems, voltage in electrical systems and temperature difference in thermal systems are similar to force and torque in mechnical systems.

- Describe the conditions which must be met for equilibrium in each of the following energy systems.

  Mechanical.
  Fluidal.
  Electrical.
  Thermal.

MECHANICAL SYSTEMS

FORCE

A linear mechanical force is a push or pull applied along a straight line, and usually is represented as an arrow indicating the direction of the applied force. Figure 1 illustrates the result when a single force is applied to an object at rest. The force causes the object ot move in the direction of the applied force. If the object is already moving when a force is applied, the force may cause it to slow down, speed up, or change direction, depending upon the direction of the applied force with respect to the original direction of motion of the object.

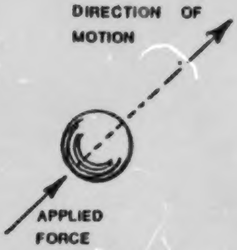


FIG. 1

In the physical world, complex systems of forces must be dealt with precisely. Figure 2 shows two forces applied to an object and a vector diagram of the forces. A vector is an arrow whose direction is the direction of the force it represents and whose length is proportional to the magnitude (strength) of the force. The resultant force is the vector sum of all individual forces. Techniques for adding forces in several directions will be discussed in later modules. The remainder of this discussion presents the basic characteristics of force, using simplified examples in which all forces act along a single line.

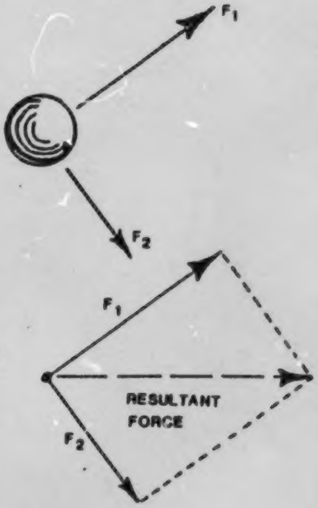


FIG. 2

The presence of forces acting upon an object does not mean necessarily that there will be a change in the state of motion of that object. A change in motion results only when the resultant force (also called the "net force," or the "differential force," and often written as ΔF) is not zero. If the net force is zero, the object remains in its original state of motion; that is, objects at rest remain at rest and objects in motion remain in motion in a straight line at a constant speed. This condition is called equilibrium.

In Figure 3 a skydiver has jumped from a plane. He falls toward the earth at an increasing speed because of the force of gravity exerted upon his body by the earth. He eventually reaches a steady maximum speed called "terminal velocity," which occurs when the upward resistance force of the air (drag) is equal to the downward pull of gravity (weight). The skydiver then continues to fall at a constant velocity in a state of equilibrium. When the parachute opens, the much larger area of the parachute causes a greater upward force due to increased wind resistance. This force slows the skydiver until a new equilibrium is reached at a lower speed and the skydiver lands safely.

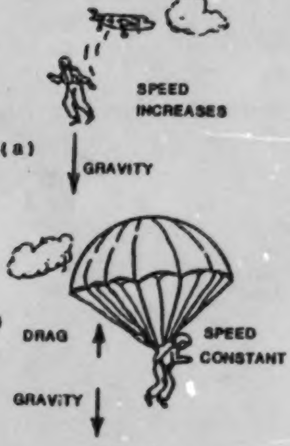


FIG. 3

The desirability often arises for finding the resultant force when several forces act within a system. For systems in which all forces lie along the same line this calculation is accomplished by assigning arbitrarily one direction along that line to be positive and the other to be negative, and then by adding all the forces as indicated by Equation 1.

$$F_T = F_1 + F_2 + F_3 + \ldots \qquad (1)$$

where

F_T = total force (resultant).

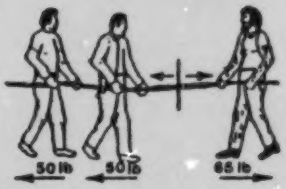
F_1, F_2, F_3... are individual forces acting on the system.

Example A illustrates the use of this equation in solving a simple problem.

EXAMPLE A : TUG-OF-WAR PROBLEM

Given: Three people are playing tug-of-war. Two people are on one side and each is pulling with a force of fifty pounds. The person on the other side is pulling with a force of sixty-five pounds.

Find: The net force and the initial direction of motion.



Solution: Define right to be positive and left to be negative. The forces acting on the rope are:

-50 lb
-50 lb
+65 lb

$$\text{Equation 1:}\quad F_T = F_1 + F_2 + F_3$$
$$= (-50\text{ lb}) + (-50\text{ lb}) + (65\text{ lb})$$
$$= -100 + 65\text{ lb}$$
$$F_T = -35\text{ lb}$$

The net force is 35 lb to the left, and the motion will be in that direction.

UNITS OF MASS AND FORCE

Mass is a measure of the quantity of matter present in an object and is a characteristic of that object regardless of its environment. Weight is the force of gravitational attraction of an object to the earth and changes as the distance from the earth changes.

The English system of units is based upon length in feet, time in seconds and force in pounds. The weight of an object in pounds (lb) is the gravitational force exerted upon that object by the earth. The English unit of mass is the slug, a derived unit defined as the amount of mass that will experience a change in speed of one foot per second when a one-pound force is applied to it for one second. The slug is not a common unit; it is defined here only to aid in the comparison of English and SI units. When English units are used, an object's weight (at the earth's surface) usually is specified in pounds.

International System of Units (SI) is based upon length in meters, time in seconds and mass in kilograms. The SI unit of force is a derived unit called a Newton (N). A Newton is defined as the amount of force that will change the speed of a one-kilogram mass by one meter per second if applied for one second. Since weight is a force, weight is measured in the SI system in Newtons. When SI units are used, an object's mass usually is specified in kilograms.

Table I shows the relationships of the units of length, time, mass and force in the SI and English systems.

TABLE 1. SI AND ENGLISH UNITS

	Length	Time	Mass	Force
SI System of Units	meter m	second s	kilogram kg	Newton * $N = \frac{kg \cdot m}{s^2}$
English System of Units	foot ft	second s	slug (no sumbol) * $\frac{lb \cdot s^2}{ft}$	pound lb

* Derived units

The weight of an object is the product of its mass and the acceleration of gravity as given in Equation 2.

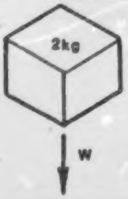
$$W = mg \qquad (2)$$

where

W = weight (a force in N or lb).
m = mass (in kg or slugs).
g = acceleration of gravity (SI: $9.8\ m/s^2$; English: $32\ ft/s^2$).

The acceleration of gravity is the acceleration that an object will experience due to the earth's attraction if it is released near the earth's surface and if no other forces act on it. An acceleration of $9.8\ m/s^2$ means that the velocity of the object will increase by 9.8 m/s during each second it falls. Example B illustrates the use of this equation in determining the weight of an object if the mass is known.

EXAMPLE B: CALCULATION OF WEIGHT



Given: A brick with a mass of 2 kg.

Find: Weight of the brick.

Solution:

$$\begin{aligned} W &= mg \\ &= (2\ kg)(9.8\ m/s^2) \\ &= 19.6\ kg \cdot m/s^2 \\ W &= 19.6\ N \end{aligned}$$

Table 2 lists some conversion factors that are useful when working with weight and mass.

TABLE 2. WEIGHT AND MASS CONVERSION FACTORS
1 lb (force) = 4.45 N (force)
1 slug (mass) = 7.23 kg (mass)
1 kg weighs 9.8 N or 2.2 lb
1 slug weighs 32.2 lb or 143 N

FORCELIKE QUANTITIES

Force is the "prime mover" in a translational mechanical system. Any change in the state of motion of an object occurs because of forces acting upon that object. A force causes the displacement of a mass through a distance.

"Forcelike quantities" are those physical quantities in other energy systems (mechanical rotational, fluidal, electrical, thermal) that act as the prime movers in those systems. In any energy system, an unbalanced forcelike quantity causes a change (a displacement of some quantity) in that system. Although these forcelike quantities do not act in their respective systems in exactly the same way that force acts in a translational mechanical system, all have certain characteristics in common with force. Force was chosen as the first prime mover to be discussed because it is the most basic. Other forcelike quantities are actually the results of forces acting within a system. The remainder of this module discusses forcelike quantities in other energy systems and begins a unification of physics by pointing out analogies that relate these quantities to the more familiar and basic concept of force.

TORQUE

Another mechanical system of interest is the mechanical rotation system. The prime mover, or forcelike quantity, in this system is torque. Torque causes a change in the state of rotational motion about an axis, just as force causes a change in the state of linear motion. Torque results when a force is applied at some distance from an axis of rotation, as shown in Figure 4. The perpendicular distance from the line of action of the

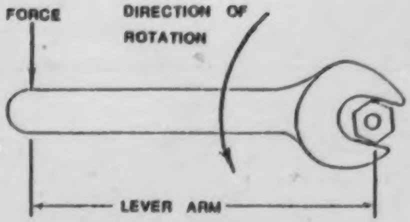


FIG. 4

force to the axis is called the "lever arm," or the "moment arm," of the force. Torque is defined by Equation 3.

$$\tau = F\ell \tag{3}$$

where

τ = torque (ft·lb or N·m).
F = force (lb or N).
ℓ = lever arm (ft or m).

The use of this equation in determining torque is illustrated in Example C.

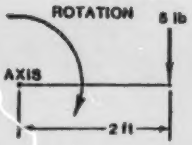
EXAMPLE C : CALCULATION OF TORQUE

Given: A force of 5 lb (22.25 N) acts with a lever arm of 2 ft (0.61 m).

Find: Torque.

Solution: English

$$\tau = F\ell$$
$$= (5\text{ lb})(2\text{ ft})$$
$$\tau = 10\text{ ft}\cdot\text{lb}$$



SI

$$\tau = F\ell$$
$$= (22.25\text{ N})(0.61\text{ m})$$
$$\tau = 13.57\text{ N}\cdot\text{m}$$

Just as the direction of a force must be specified, the direction of the rotation resulting from a torque must be specified. One method of specifying torques is as clockwise (cw) and counterclockwise (ccw). When solving problems involving the total torque in a system one of these directions is chosen as positive, and the other negative and the torques are added algebraically, as indicated in Equation 4.

$$\tau_T = \tau_1 + \tau_2 + \tau_3 + \ldots \tag{4}$$

where

τ_T = total torque.

τ_2, τ_2, τ_3 ... are individual torques acting on the system.

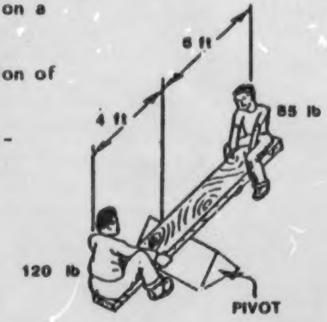
Examples D and E show the use of this equation and of Equation 3 in solving torque problems.

EXAMPLE D: BEAM PROBLEM

Given: Two people are sitting on a beam as shown.

Find: Total torque and direction of rotation.

Solution: Define clockwise as positive. Torque of person on right:



$$\tau_R = F_R \ell_R$$
$$= (85 \text{ lb})(6 \text{ ft}) \text{ cw}$$
$$= 510 \text{ ft·lb cw}$$
$$\tau_R = 510 \text{ ft·lb}$$

Torque of person on left:

$$\tau_L = F_L \ell_L$$
$$= (120 \text{ lb})(4 \text{ ft}) \text{ ccw}$$
$$= 480 \text{ ft·lb ccw}$$
$$\tau_L = -480 \text{ ft·lb}$$

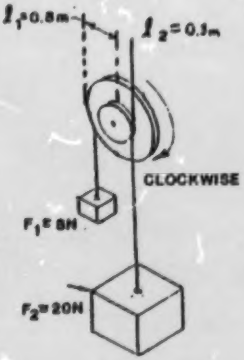
Total torque:

$$\tau_T = \tau_R + \tau_L$$
$$= 510 \text{ ft·lb} + (-480 \text{ ft·lb})$$
$$\tau_T = 30 \text{ ft lb·cw}$$

The beam will rotate in a clockwise direction, with a torque of 30 ft·lb. The 120 lb person will be lifted, the 85 lb person lowered.

EXAMPLE E: PULLEY SYSTEM PROBLEM

Given: A pulley system has weights attached to provide forces as shown.



Find: Total torque and direction of rotation.

Solution: Define clockwise as positive:

$$\begin{aligned} \tau_1 &= F\,\ell \\ \tau_1\ (ccw) &= -F_1\,\ell_1 \\ &= -(8\ N)(0.8\ m) \\ \tau_1 &= -6.4\ N{\cdot}m \\ \tau_2\ (cw) &= F \\ &= (20\ N)(0.3\ m) \\ \tau_2 &= 6.0\ N{\cdot}m \\ \tau_T &= \tau_1 + \tau_2 \\ &= -6.4\ N{\cdot}m + 6.0\ N{\cdot}m \\ \tau_T &= -0.4\ N{\cdot}m \end{aligned}$$

The pulley system will turn in a counterclockwise direction with a torque of 0.4 N·m.

If the total torque on any system is zero, that system is in rotational equilibrium and its rotational rate remains constant.

FLUIDAL SYSTEMS

The prime mover, or forcelike quantity, in fluidal systems (hydraulic or pneumatic) is called "pressure." Pressure is defined as the force per unit area exerted by a fluid and is given by Equation 5.

$$p = \frac{F}{A} \tag{5}$$

where

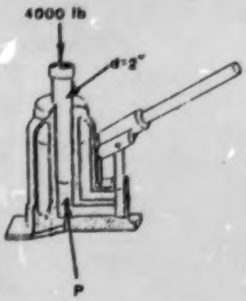
p = pressure (lb/in^2 or N/m^2).
F = force (lb or N).
A = area (in^2 or m^2).

Examples F and G illustrate the use of Equation 5 in solving two types of problems.

EXAMPLE F: FLUID PRESSURE IN A HYDRAULIC JACK

Given: A hydraulic jack is rated at 4000 lb lifting capacity and has a large piston with a diameter of 2 inches.

Find: Fluid pressure in the jack at maximum load.



Solution: Area: $A = \pi r^2$

$$= (3.14)(1\ \text{in})^2$$
$$A = 3.14\ \text{in}^2$$
$$p = \frac{F}{A}$$
$$= (4000\ \text{lb}) \div (3.14\ \text{in}^2)$$
$$p = 1274\ \text{lb/in}^2$$

EXAMPLE G: FORCE ON AIRPLANE WINDOW

Given: An airplane window with surface area of 2 square feet. Pressure inside the cabin is 8 pounds per square inch greater than the pressure outside.

Find: The net force on the window.

Solution:

$$p = \frac{F}{A}$$
$$F = pA$$
$$F = (8\ \frac{\text{lb}}{\text{in}^2})(288\ \text{in}^2) \qquad 1\ \text{ft}^2 = 144\ \text{in}^2$$
$$\text{Force} = 2304\ \text{lb (outward)}$$

The window must withstand a net force of 2304 lb.

Pressure of a gas in a container depends upon the number and speed of molecules coliding against the sides of the container. Therefore, one way to increase the pressure on a gas in a container is either to pump more molecules into the container or to force the same molecules into a small container. To decrease the pressure, some of the molecules can be released from the container or placed into a larger container. Another way to increase pressure is to heat a fluid inside a closed container. This procedure causes the molecules to increase their speed, creating an increased pressure. Cooling of the container has the reverse effect on the molecules and, consequently, decreases the pressure.

Pressure in a liquid increases as the depth increases. For instance, if water in a tank is made deeper, there will be more water above the location at which pressure is measured; therefore, the weight of the additional water will create a greater pressure at the deeper position. The pressure at any depth in a container filled with a liquid depends upon the height of the liquid above that point, the density of the liquid and the acceleration of gravity, as given in Equation 6.

$$p = \rho g h \tag{6}$$

where

p = pressure (N/m^2).
ρ = density (kg/m^3).
h = height (m).

In the English system of units this is usually written in the form of Equation 7.

$$p = \rho_w h \tag{7}$$

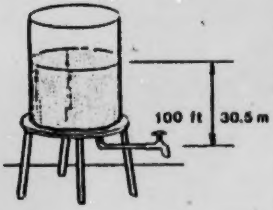
where

ρ_w = weight density in lb/ft^3.
h = height in feet.

Weight density refers to the weight (gravitational force) of a volume of liquid, and contains the gravitational constant as a factor. Example H illustrates the use of Equations 6 and 7 in solving a problem.

EXAMPLE H: WATER PRESSURE PROBLEM

Given: The height of the water in a water system is 100 ft (30.5 m) above a faucet. The density of water (SI units) is 1000 kg/m^3. The weight density of water (English units) is 62.4 lb/ft^3.



Find: The pressure at the faucet in SI and English units.

Solution: (a) SI Units:

$$p = \rho g h$$
$$= (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(30.5\ \text{m})$$

$$p = 2.99 \times 10^5 \text{ kg/m}\cdot\text{s}^2$$
$$p = 2.99 \times 10^5 \text{ N/m}^2$$

Note that $\frac{N}{m^2} = \frac{(\frac{kg\cdot m}{s^2})}{m^2} = \frac{kg}{m\cdot s^2}$

The units of pressure are force per area.

(b) English Units: $p = \rho_w h$

$$= (62.4 \text{ lb/ft}^3)(100 \text{ ft})$$
$$= 6240 \text{ lb/ft}^2$$
$$= (6240 \text{ lb/ft}^2)(\frac{1 \text{ ft}^2}{144 \text{ in}^2})$$
$$p = 43.3 \text{ lb/in}^2$$

The units have been converted to the more familiar pounds per square inch. This is also force per unit area.

Motion occurs in a fluid as the result of a pressure difference. For this reason, pressure plays the role of the forcelike quantity in a fluidal system. Figure 5 shows two tanks with different liquid levels connected by a pipe with a valve. The pressures p_1 and p_2 at the ends of the pipe depend upon the height of liquid above that point in each tank. Since the liquid in tank 2 is higher, p_2 is greater than p_1. When the value is opened, this pressure differential will cause liquid to flow through the pipe until the liquid levels are the same in both tanks. As this point, p_1 and p_2 will be equal and the system will be in equilibrium. Equilibrium depends only upon the heights being equal, not upon the volume of liquid in the tanks. The system shown in Figure 6 is in equilibrium, although the right tank contains more liquid.

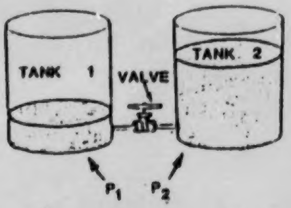


FIG. 5

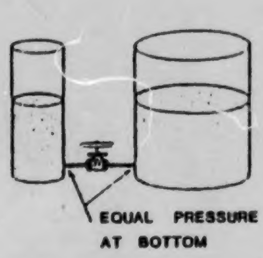


FIG. 6

ELECTRICAL SYSTEMS

The prime mover, or forcelike quantity, in electrical systems is the voltage, sometimes referred to as "potential difference" or "electromotive force." Voltage occurs in an electrical system because of the

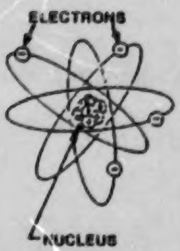


FIG. 7

separation of electrical charge in the system.

The origin of charge lies in the atomic structure of matter. All matter is composed of atoms. The structure of an atom is represented in Figure 7. The nucleus contains most of the atomic mass in the form of neutrons and protons. The protons have a positive charge, while the neutrons are neutral (no charge). The nucleus is surrounded by a cloud composed of the much less massive electrons. Each electron has a negative charge of the same magnitude as the positive charge of a proton. A normal atom is neutral (has no net charge) because it has the same number of electrons and protons. Electrons are bound to the atom by electrical forces of attraction that obey the rule: unlike charges attract and like charges repel. These forces result in a voltage whenever the positive and negative charges are separated. Four common ways of creating a potential difference are listed in Table 3.

TABLE 3. METHODS USED TO PRODUCE A POTENTIAL DIFFERENCE

Friction	Rubbing a cat's fur, for example, can create static electricity. Friction strips away electrons from the cat's fur, leaving a positive charge on the fur and a negative charge on the hand.
Chemical	Chemical energy in dry cells (flashlight battery type) or in wet cells (auto battery type) separates the charge to form a positive pole and a negative pole.
Magnetic Mechanical	A coil of wire moving in a magnetic field produces a force on electric charges that separates the positive and negative charges.
Light	Light energy falling on certain materials causes the separation of electrons from their atoms. Light meters make use of this process and indirectly record light intensities.

"Neutral" objects are those that contain the same quantity of positive and negative charge.

In Figure 8 some of the electrons have been moved from one object to another. The object with excess electrons is said to have a negative charge and the one from which the electrons have been removed, therefore, has a positive charge. If a path is provided for the electrons from one object to the other, an electric current will flow; that is, charge will be transferred from one object to the other until both have the same electrical potential. Potential difference moves charge in an electrical system in the same way that pressure moves fluid in a fluidal system. Thus, potential difference plays the role of a forcelike quantity in electrical systems.

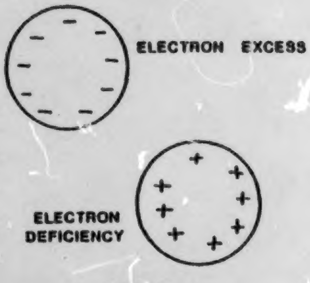
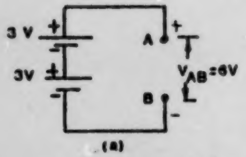


FIG. 8

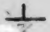
Electric current is the rate at which charge moves through a system. Current exists in a circuit whenever two points of different electrical potential are connected by an electrical conductor. If there is no potential difference, the system is in equilibrium, and no charge flows.

The potential difference between any two points in an electrical system is measured in volts (V). If a circuit contains more than one voltage source, the total voltage is the sum of the individual sources. In Figure 9a, two batteries are connected in series, with the positive terminal of one connected to the negative terminal of the other. In this circuit, both voltage sources will cause charge flow in the same direction if the circuit is completed. The total voltage produced is the sum of the individual voltages. In this case, a net potential difference acts as a forcelike quantity to cause charge flow.

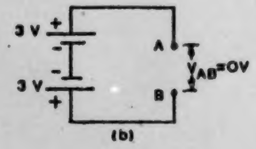
In Figure 9b, the lower battery has been reversed. In this circuit, the two voltages oppose each other, and the total voltage is their difference, in this case zero. This zero value means that no charge will flow if points A and B are connected. The absence of a net potential difference means that no forcelike quantity is present to cause charge flow.



(a)



SCHEMATIC SYMBOL FOR BATTERY



(b)

FIG. 9

THERMAL SYSTEMS

The prime mover, or forcelike quantity, in thermal systems is temperature difference, often written as ΔT. Heat energy always flows naturally from warmer regions to colder regions; and the rate of heat flow in any system is dependent upon temperature difference, just as fluid flow is dependent upon pressure difference, and electric current is dependent upon voltage difference.

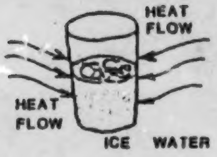


FIG. 10

A glass of water sitting in a warm room (Figure 10) will gain heat energy from its surroundings until all the ice melts and the water reaches the same temperature as the room. No further heat energy will flow in the final equilibrium condition, because no temperature difference exists.

All matter is composed of molecules (single atoms or groups of atoms) that are in constant motion. Heat energy is the energy of motion (kinetic energy) of moving molecules. This energy is transferred through a material by the action of the forces that exist between molecules of that material.

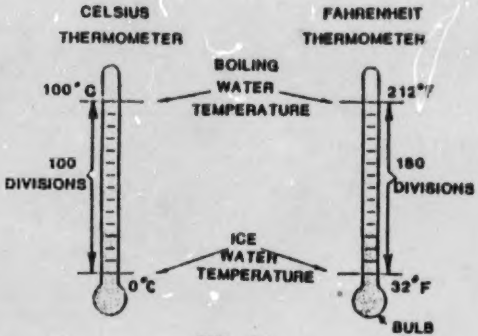


FIG. 11

Temperature is a measure of the average kinetic energy of the molecules of a substance. The two most commonly-used temperature scales are shown in Figure 11. The Fahrenheit scale is a part of the English system of units; and the Celsius scale, sometimes called "centigrade," is a part of the SI system. Both systems use the freezing and boiling points of water as references, and divide the temperature difference between these points into an arbitrary number of divisions called degrees. Table 4 lists the formulas for conversion of temperature in one system to equivalent temperature in the other.

TABLE 4 . TEMPERATURE CONVERSION

Conversion of T°C to T°F:

$$T°F = 32°F + 9/5\ T°C$$

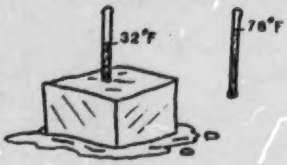
Conversion of T°F to T°C:

$$T°C = 5/9\ (T°F - 32°F)$$

When only the temperature difference is important, and not the actual temperature reading, conversion is made using the relationship 9F° = 5C°, or 1.8F° = 1C°. Temperatures are specified in degrees Celsius (°C) or degrees Fahrenheit (°F). Temperature difference is expressed in Celsius degrees (C°) or Fahrenheit degrees (F°), as shown in Example 1.

EXAMPLE 1: TEMPERATURE CONVERSION

Given: A block of ice at 32°F is left in a room at 78°F.



Find:

(a) Temperature difference in Fahrenheit degrees.

(b) Temperature difference in Celsius degrees.

(c) Room temperature in degrees Celsius.

Solution:

(a) $\Delta T = T_2 - T_1$

$= 78°F - 32°F$

$\Delta T = 46F°$.

The temperature difference is 46 Fahrenheit degrees.

(b) $\Delta T\ (C°) = 5/9\ \Delta T\ F°$

$= 5/9\ (46F°)$

$\Delta T = 25.5C°$

The temperature difference is 25.5 Celsius degrees.

(c) $T\ (°C) = 5/9\ (78°F - 32°F)$

$= 5/9\ (46°F)$

$T = 25.5°C$

The temperature of the room is 25.5 degrees Celsius.

In thermal systems, temperature difference causes heat flow; therefore, temperature acts as a forcelike quantity.

SUMMARY

Forcelike quantities are the "prime movers" in all physical systems. Although one would be incorrect to say that net force, net torque, pressure differential, voltage, and temperature difference affect their respective energy systems in exactly the same way, they play strikingly analogous roles. The basic and very important similarities of these physical quantities can be of value in applying knowledge gained in one energy system to the solution of analogous problems in other energy systems. Table 5 summarizes the forcelike quantities in the four energy systems.

TABLE 5. FORCELIKE QUANTITIES

Energy System	Forcelike Quantity	Units		Quantity Moved
		English	SI	
MECHANICAL				
Translational	Force	lb	N	Mass moved through a distance
Rotational	Torque	ft·lb	N·m	Mass rotated through an angle
FLUIDAL	Pressure	lb/in^2	N/m^2	Fluid volume
ELECTRICAL	Potential Difference		V	Charge
THERMAL	Temperature Difference	F°	C°	Heat energy

26

MECHANICAL SYSTEMS, LINEAR MOTION

MATERIALS

Wood block (may substitute metal or plastic block) approximately 2" x 4" x 8" with cup hook.
Spring balance (pound or ounces and Newton calibration) with appropriate scale range.
Assorted weights (typical range) and weight hanger.
String.
Pulley and bracket for table mount.

PROCEDURE

This experiment demonstrates that forces in mechanics, when applied to objects, cause motion. Such occurrences are very common in our everyday lives, so nothing very startling is being demonstrated here. But, since the necessity arises to compare forcelike quantities in other energy systems, it is well to make a few simple observations concerning forces applied to masses. At the same time, the student will practice measuring forces and describing these measurements in appropriate units.

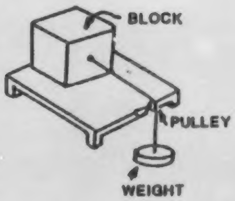


FIG. 12

1. Arrange the block, pulley and weight hangers as shown in Figure 12. Add gram weights to weight pan in small amounts until the block just begins to move (but not speed up) at a constant speed across table. A slight push may be necessary to start motion. Record weight in Data Table 1.

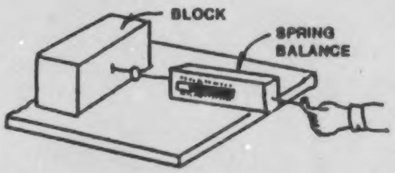


FIG. 13

2. Arrange block and spring balance as shown in Figure 13. First, use a spring balance calibrated in pounds or ounces, then one in Newtons. Pull on spring balance carefully until block just begins to move at a constant speed. A slightly larger force may be required to start the motion than to maintain the motion at a constant speed. Record only the force necessary to produce a constant speed. Try this several times and record the average read-

ings (lb and Newtons) in Data Table 1.

3. Mathematically convert the force due to the gram weights (weights on weight hanger) to Newtons and the spring balance reading in pounds to Newtons. Record these values in Data Table 1. Calculate the percent difference between each of the forces in Newtons. Percent difference should be within limits specified by the instructor.

DATA TABLE 1. MECHANICAL LINEAR FORCES

Method Used	Measured Force	Converted Value in Newtons	% Difference
1. Block and weight on pan (gf)	______gf	______N	(1-2)_____%
2. Block and spring balance (lbf)	______lbf	______N	(2-3)_____%
3. Block and spring balance (Newtons)	______N		(1-3)_____%

MECHANICAL SYSTEM, ROTATIONAL MOTION

MATERIALS

Torque wrench (typical: 0 - 150 ft·lb).
Bolt and nut (typical: 3/8" or 1/2" diameter).
Socket for torque wrench and open-end wrench to fit bolt and nut.
Vice or other mounting device for bolt.

PROCEDURE

This experiment demonstrates the effect of applying a torque to a mechanical system. It shows that torque produces angular displacement, or rotational motion, and demonstrates that torque is the forcelike quantity in rotational motion.

1. Place a bolt in a vice or other suitable holder, and tighten a nut on the bolt to 20 ft·lb or more, using a torque wrench as illustrated in Figure 14. Loosen nut using torque wrench. Record torque wrench reading when nut is tightened and when it is loosened.

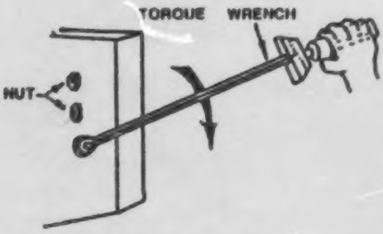


FIG. 14

2. Again, tighten nut on bolt carefully to same torque reading as in number 1 above. Attach spring balance (which measures in pounds) to a wrench at a measured distance from center of the bolt, as in Figure 15. Record distance in feet. Pull on spring balance to loosen nut. Record reading on spring balance when nut is loosened.

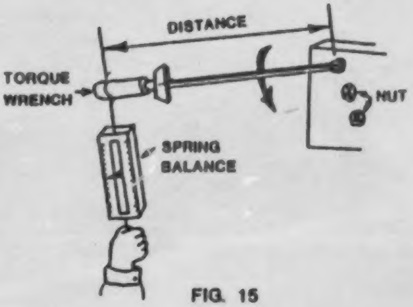


FIG. 15

3. Calculate torque value, and compare this value with value given on torque wrench before determining the percent difference between the two values. Percent difference should be within limits specified by the instructor. Record these values in Data Table 2.

DATA TABLE 2. MECHANICAL ROTATIONAL TORQUE

1. Torque to tighten nut __________ ft·lb
   Torque to loosen nut __________ ft·lb

2. Torque to tighten nut __________ ft·lb
   (reading on torque wrench)
   Length of handle __________ ft
   Spring balance reading __________ lb
   Calculated torque __________ ft·lb
   % Difference = __________ %
   $\% \text{ Difference} = \frac{\text{calculated} - \text{actual}}{\text{calculated}} \times 100$

FLUIDAL SYSTEM

MATERIALS

Meter stick
U-tube manometer (approximately 50 cm high).

PROCEDURE

In the Discussion section of this module, pressure or pressure difference was introduced as the forcelike quantity in fluid systems. In the following experiment, the student will produce a pressure difference on a simple fluid system and will measure the pressure difference.

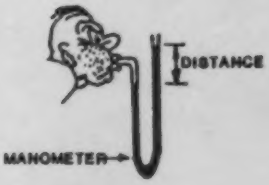


FIG. 16

1. Partially fill U-tube with water. Blow one end of tube (or use a squeeze bulb), and measure the maximum difference in levels of water (in inches), as illustrated in Figure 16.
2. The measured difference in levels is water pressure (in inches of water) that was produced by blowing. Convert value of pressure from inches of water to lb/in^2 (1 lb/in^2 is equal to 27.7 in of water). Record values in Data Table 3.

DATA TABLE 3. FLUID PRESSURE

Pressure difference produced by blowing ________ in of water ________ lb/in^2

THERMAL SYSTEM

MATERIALS

Metal rod (typical 1/8" diameter x 12" copper).
Bunsen burner (may substitute electric hot plate).
Temperature stick (crayola can be used).
Stand or fixture to hold metal rod.

PROCEDURE

In thermal systems, a temperature difference serves as a forcelike quantity to cause heat energy to flow from regions of high temperature to regions of low temperature. In this experiment, a temperature difference is created in a metal rod when heat is applied to one end of the rod. The rod has been lined with wax material from a temperature stick. One then can "see" the heat flowing away from the hot end as the coating from the temperature stick changes its composition.

1. Draw a line with temperature stick along length of metal rod, and fasten rod in stand.

2. Bunsen burner at free end of rod, and adjust it so that the rod is about 3 cm above the Bunsen burner, as illustrated in Figure 17.

3. Light burner, and observe line from the heat stick melt progressively from hot zone to cold end. This melting illustrates the movement of heat down the length of the rod. At some point melting of the line may stop because heat in the rod is being drawn off by cold clamp and stand faster than it can be used to melt wax material on the outside of the rod.

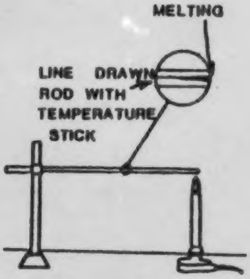


FIG. 17

4. Answer the following question:

   - How far from the Bunsen burner did the heat flow?_______.
   - In which direction does heat always flow? _________.
   - What forcelike quantity caused the flow of heat down the length of the rod? ________.

ELECTRICAL SYSTEM

MATERIALS

6. V d.c. source.
Capacitor (typical 1μF).
Voltmeter (0 - 10 V range, typical calibration).
Two alligator clip leads for voltmeter.

PROCEDURE

In the electrical system, voltage difference across battery terminals serves as the forcelike quantity, forcing charge to move when the positive and negative terminals are connected. Although movement of electrical charge is invisible, this experiment will show that a potential difference does move charge when connected in a circuit.

1. Use voltmeter to measure emf of battery. Record reading in Data Table 4. Also check capacitor with voltmeter, and record this reading. The capacitor reading should be zero. Next, connect leads from battery to capacitor; wait a few seconds, and disconnect leads. (What has the connection of the battery to the capacitor accomplished?)

2. Attach leads from voltmeter to capacitor, and record initial voltage reading. Watch voltage decay. This represents movement of charge from negative side of capacitor to the positive side. Record these readings in Data Table 4.

DATA TABLE 4. POTENTIAL DIFFERENCE

Potential Difference Before Charging Capacitor	Potential Difference After Charging Capacitor
Battery emf _________volts	Initial reading _______volts
Capacitor emf ________volts	Final reading ________volts

What showed that charge moved in the capacitor during the decrease in voltage? ____

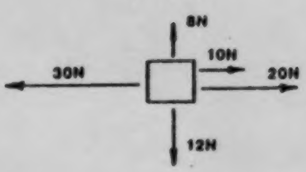
SELF CHECK

1. Define the following physical quantities; and, where applicable, state their units in both the SI and English systems.

   a. Force.
   b. Torque.
   c. Pressure.
   d. Voltage.
   e. Temperature difference.

2. An object is acted upon by three simultaneous forces: 40 lb to the right, 180 lb to the left, and 200 lb to the right. Determine the resultant force and the direction of motion.

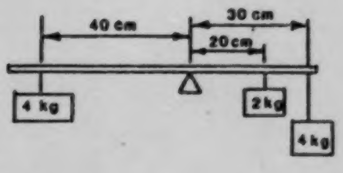
3. The diagram at the right shows five forces acting upon an object initially at rest. Determine the net force and the direction of motion.



4. A force of 10 lb is used to produce a torque of 32.8 ft·lb. Determine the lever arm.

5. Convert the values in problem 4 to metric units, and solve for lever arm in meters. Compare answer to that from problem 4.

6. Three masses are hung from a meter stick, as shown at the right. Find the net torque and the direction of rotation.



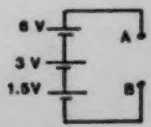
7. A 130 lb lady wearing spike heel shoes, with heels measuring 1/2" x 1/2", is standing on a linoleum floor. Calculate how much pressure will be applied to the linoleum if she puts all her weight on one heel.

8. Calculate the water pressure in pounds per square inch at the bottom of a lake 250 ft deep. (Weight density of water is 62.4 lb/ft^3.)

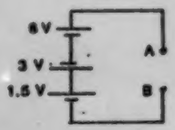
9. Calculate the depth of water in a tank if the pressure at the bottom is 10^5 N/m^2. (Density of water is 1000 kg/m^2.)

10. Determine the total voltage V_{AB} in each of the following circuits:

(A)



(B)



11. Convert each of the following temperatures in °F to temperatures in °C.

a. 413°F.
b. 26°F.
c. -13°F.

12. Convert the following temperatures in °C to temperatures in °F.

a. 25.5°C.

b. 5°C.
c. -40°C.

13. Describe in a short paragraph the similarities of forcelike quantities in the four energy systems discussed in this module.

14. Describe the conditions necessary for equilibrium in each energy system.

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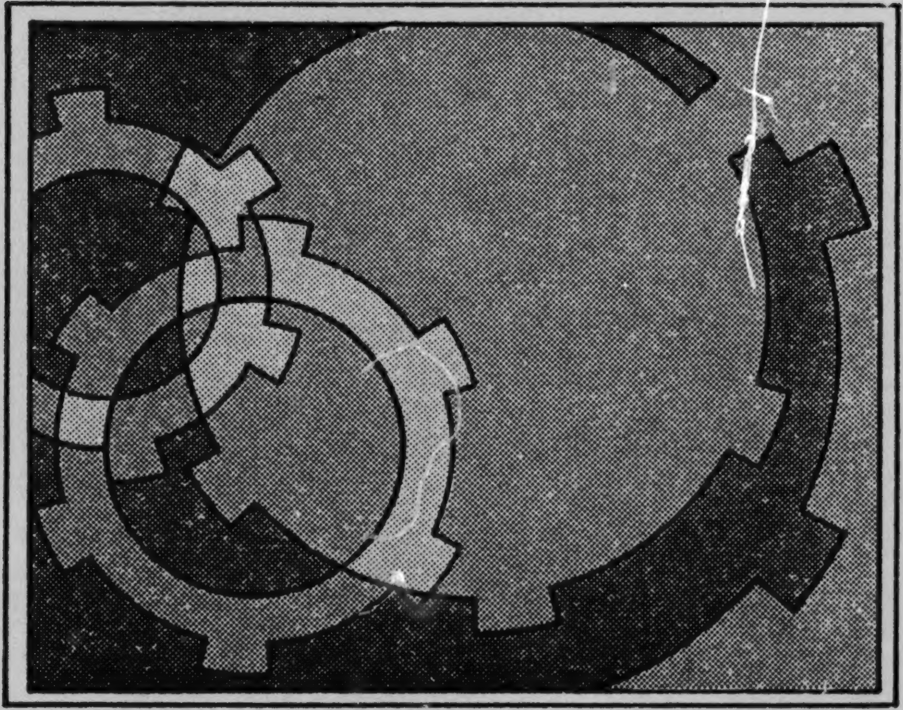
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 1 M 1

GRAVITY



MECHANICAL APPLICATION OF MOMENTUM

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Gravitational force is exerted by each object in the universe upon all other objects. The force of gravity upon an object is weight — not to be confused with mass, which is the resistance of an object to a change in its motion. Gravity influences man and machinery upon earth and in space. The relationship between gravitational force and acceleration of free-falling objects is discussed in this module. In the laboratory, the student will determine both the weight and the mass of several objects.

PREREQUISITES

Student should have completed Concept Module I-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define Newton's "law of universal gravitation."
- Explain the difference between weight and mass.
- Calculate the gravitational attraction of one object upon another, given their mass and separation distance.
- Describe the inverse-square law.
- Calculate the velocity of an object in free fall, given the time of the fall.
- Given the appropriate equipment, measure the weight of several objects by two methods; and calculate their mass.

DISCUSSION

THE LAW OF UNIVERSAL GRAVITATION

Newton's "law of universal gravitation" states, "Every object in the universe attracts every other object with a force directly proportional to each of their masses and inversely proportional to the square of the distance separating them."

If a ball is thrown upward, it is pulled back to earth by the gravitational force exerted by the earth upon the ball. Also, the earth itself moves around the sun in an elliptical orbit because of the gravitational force which the sun exerts upon it; otherwise, the earth would move away in a straight line.

Newton was the first person to conclude that the same force that causes the moon to move about the earth also causes an apple to fall from a tree to the earth. The moon also is falling toward the earth; otherwise, it would move away in a straight line. It falls just enough each second to remain in its orbit around the earth, as illustrated in Figure 1. From this original insight, Newton was able to explain the motion of the planets about the sun and of the moon about the earth.

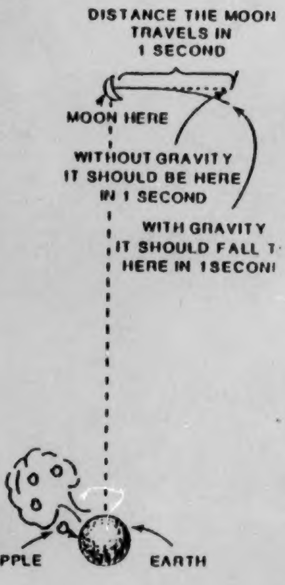

DISTANCE THE MOON TRAVELS IN 1 SECOND
MOON HERE
WITHOUT GRAVITY IT SHOULD BE HERE IN 1 SECOND
WITH GRAVITY IT SHOULD FALL T
HERE IN 1SECON
APPLE
EARTH

FIG. 1

WEIGHT AND MASS

An important distinction exists between weight and mass. Weight is "the force due to gravity upon a body" and depends upon its location relative to an astronomical body. An object would weigh more upon earth than upon the moon, for example. Mass, however, is the same for an object everywhere in the universe and can be defined as "the quantity of matter in a body, or the property of a body's resistance to change of motion." Mass, is thus a measure of the inertia, or sluggishness, of an object in stopping or changing its motion where there is no friction. An object weighs less upon the moon than upon the earth, yet its mass is the same.

FORCE OF GRAVITY

Newton's law of gravitation, or gravitational force, is expressed in terms of the masses of the objects involved and their separation distance, indicated by Equation 1.

$$F_g = \frac{Gm_1m_2}{d^2} \qquad (1)$$

where

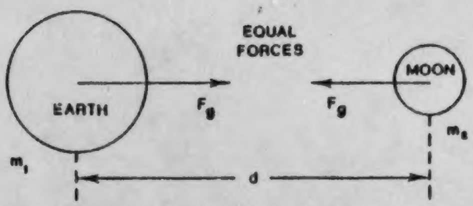
F_g = force of gravity.

G = universal gravitational proportional constant, 6.67×10^{-11} m³/kg - s².

m_1 = mass of object 1.

m_2 = mass of object 2.

d = distance between objects.



(d MEASURED FROM CENTER OF BOTH BODIES)

FIG. 2

Equation 1 can be used to explain the motion of all celestial bodies in our solar system. Figure 2 illustrates the gravitational force between two (approximately) spherical bodies, the earth and the moon. The distance between them is measured from their centers. The force of gravitational attraction upon one sphere is equal and opposite to that upon the other. Equation 1 is used in solving a problem in Example A.

EXAMPLE A: GRAVITATIONAL FORCE

Given: A ball with a mass of 50 kg is located at the surface of the earth (which has a radius of $d = 6.4 \times 10^6$ meters and a mass of $m_e = 6.02 \times 10^{24}$ kg).

Find: The force the earth exerts upon the ball and the force the ball exerts upon the earth.

Solution:

$$F_g = \frac{Gm_1m_2}{d^2}$$

$$= \frac{(6.67 \times 10^{11}\ \text{m}^3/\text{kg s}^2)(6.02 \times 10^{24}\ \text{kg})(50\ \text{kg})}{(6.4 \times 10^6\ \text{m})^2}$$

$$F_g = 490 \text{ Newtons}$$

The force that the earth exerts upon the ball is equal to the force the ball exerts upon the earth; they are oppositely directed.

Figures 3 and 4 illustrate how gravitational force changes with separation distance. Within a planet of uniform composition and density, gravitational force is directly proportional to the radial distance from the center and is maximum at its surface. Because of the distance squared factor in the denominator, however, the force from the surface outward decreases rapidly as the distance increases (Figure 3). This relationship is called the "inverse-square law," and means that the earth's gravitational pull upon an outside object varies inversely with the square of its distance from the center of the earth. For example, an object that weighs one pound at the surface of the earth will weigh 1/4 pound when it is twice as far from the center of the earth (Figure 4).

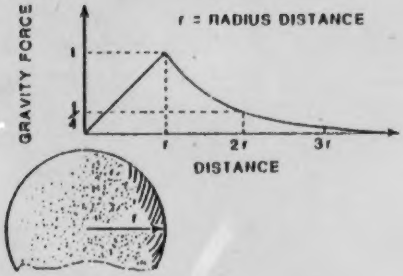


FIG. 3

Gravitational force is a relatively weak force: A mass the size of the earth exerts a force of only 150 lb upon a person weighing 150 lb. Two people cannot detect or measure their mutual gravitational attraction, no matter how close they might get. Other forces in nature, such as the electrical force, are much stronger than the gravitational force.

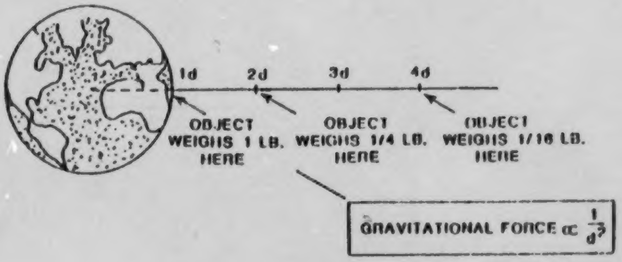

1d
2d
3d
4d
OBJECT WEIGHS 1 LB. HERE
OBJECT WEIGHS 1/4 LB. HERE
OBJECT WEIGHS 1/16 LB. HERE
GRAVITATIONAL FORCE $\propto \frac{1}{d^2}$

FIG. 4

The gravitational force equation (Equation 1) for objects on the surface of the earth can be rearranged as in Equation 2.

$$F_g = mg \qquad (2)$$

where

F_g = force of gravity.

m = mass of object on surface.

$g = \frac{Gm_e}{d^2}$, or acceleration due to gravity; where m_e is mass of the earth, and d is radius of the earth (4,000 miles).

That is,

$$F_g = m\left(\frac{Gm_e}{d^2}\right) = mg$$

The term g (acceleration due to gravity) is almost a constant for positions close to the surface of the earth, as shown by the following calculated values:

$g = 9.800\ m/s^2$ for $d = 4000$ mi (at the surface of the earth).
$g = 9.795\ m/s^2$ for $d = 4001$ mi (one mile above the surface).
$g = 9.751\ m/s^2$ for $d = 4010$ mi (ten miles above the surface).

The accepted value of g at the earth's surface is $9.8\ m/s^2$ in the International system of units and $32\ ft/s^2$ in the British system of units. An object would have to be high above the earth's surface for the g value to be noticeably different.

VELOCITY

If a parachutist falls at constant velocity, the downward gravitational force is balanced by the upward drag force of the parachute. Since the net force is zero, he falls at a constant velocity, or distance traveled divided by time, as indicated by Equation 3.

$$v = \frac{d}{t} \qquad (3)$$

where
v = velocity.
d = distance in feet.
t = time in seconds.

Equation 3 is utilized to solve a problem for constant velocity in Example B.

EXAMPLE B

Given: A parachutist falls 180 ft during each second.

Find: Velocity of the parachutist as he falls through the air.

Solution:
$$v = \frac{d}{t}$$
$$= \frac{180\ ft}{1\ s}$$
$$v = 180\ ft/s$$

Notice that this example illustrates constant velocity, a velocity that does not change from second to second.

ACCELERATION

If there were no drag force acting upon a falling object, its velocity would increase each second (Figure 4). An object dropped from rest, with no resistive force acting upon it, will be traveling with a velocity of 32 ft/s after the first second, 64 ft/s after the next second, 96 ft/s after the third second, etc. In other words, its velocity changes by 32 ft/s each second it falls. The acceleration of an object is defined by Equation 4.

$$a = \frac{\Delta v}{\Delta t} \qquad (4)$$

where

a = acceleration.
Δv = change in velocity.
Δt = time elapsed in seconds.

For a freely-falling object, this equation becomes $a = \frac{(32\ \text{ft/s})}{(1\ \text{s})}$

$a = 32\ \text{ft/s}^2$

Equation 4 can be arranged for velocity, as indicated by Equation 5 and Example C.

$$v = at \qquad (5)$$

where

v = velocity of free-falling body.
a = acceleration.
t = time in seconds.

EXAMPLE C

Given: An object falls for five seconds with no drag force acting upon it.

Find: The velocity of the object at the end of the five-second fall.

Solution: $v = at$

$$v = (32\ \text{ft/s}^2)(5\ \text{s})$$
$$v = 160\ \text{ft/s}$$

Since velocity increases during each second of fall for a freely-falling object, the distance of fall during each second increases. A free-falling object starting from rest has a velocity of 32 ft/s at the end of the first second, and thus an average velocity of 16 ft/s during the first second; therefore, it travels a distance of 16 ft during the first second. Figure 5 shows the distance an object falls during the first few seconds when it starts from rest. (Velocity is discussed in more detail in later modules.)

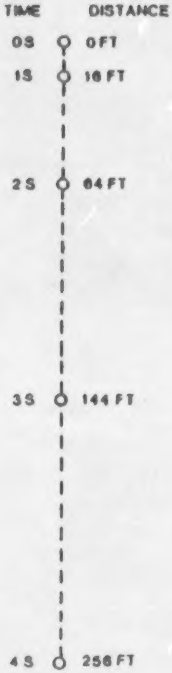


FIG. 5

FORCE ACTING UPON AN OBJECT

If the net force acting upon an object is zero, the object will have no acceleration; however, if a net force acts upon an object, the object will experience acceleration. Acceleration is proportional to force, as indicated by Equation 6.

$$F = ma \tag{6}$$

where
F = net force acting upon an object.

When the force acting upon an object is known, together with its resulting acceleration, its mass can be calculated from Equation 7.

$$m = \frac{F}{a} \tag{7}$$

The mass of an object is calculated in Example D.

EXAMPLE D

Given: An object experiences (a) an acceleration of 5 m/s^2 when a force of 10 Newtons acts upon it, and (b) an acceleration of 15 m/s^2 when a force of 30 Newtons acts upon it.

Find: Mass of the object for these conditions.

Solution: (a) $m = \frac{F}{a}$ (Recall that $\frac{1N \cdot s^2}{m} = 1$ kg.)

$$= \frac{(10\ N)}{(5\ m/s^2)}$$

$$m = 2\ kg$$

(b) $m = \frac{F}{a}$

$$= \frac{(30\ N)}{(15\ m/s^2)}$$

$$m = 2\ kg$$

Notice that the results are the same for both sets of conditions, since the same mass is involved.

Equation 2 demonstrated that the force acting upon an object due to gravity is $F_g = m\ (\frac{Gm_e}{d^2}) = mg$. This force is related to acceleration through Equation 6, $F = ma$. In these equations, acceleration, a, must exactly equal g; that is, $a = g = 32\ ft/s^2$, or $9.8\ m/s^2$, which is the acceleration due to gravity.

This equality means that the value measured for the acceleration of a free-falling object ($9.8\ m/s^2$, or $32\ ft/s^2$) is the same as the value of g calculated from the relationship $g = \frac{Gm_e}{d^2}$. Thus, each second an object falls, its velocity increases by 32 ft/s. Newton was able to show, using this result, that his law of gravity (Equation 1) gave the same value for g, the acceleration due to gravity, as the experimentally-determined value. The calculation is simply as follows:

$$g = \frac{Gm_e}{d^2}$$

$$= \frac{(6.67 \times 10^{-11}\ m^3/kg - s^2)(6.02 \times 10^{24}\ kg)}{(6.4 \times 10^6\ m)^2}$$

$$g = 9.8\ m/s^2$$

When Newton first calculated a value of g using this relationship, he obtained a result different from the experimentally-measured value; and because of this discrepancy, he filed away his papers. About twenty years later, a new measurement was made for the mass of the earth. This new value differed considerably from the previous value that Newton had used. He recalculated the value of g with the improved value of the mass of the earth and obtained results very close to the experimental value. Only then did he publish his papers on gravity and mechanics.

WEIGHT

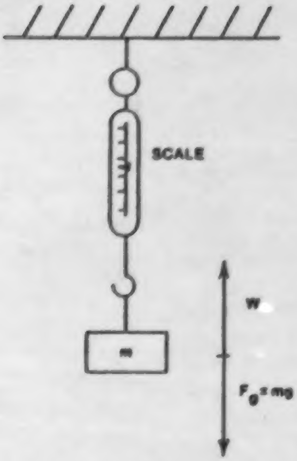


FIG. 6

The gravitational force that the earth exerts upon an object of mass m can be measured by use of a spring scale (Figure 6). Gravitational force is mg, and the upward force which the scale exerts upon the mass is W. Since the mass is not moving, the upward force minus downward gravitational force equals zero, or $W - mg = 0$. Rearranged, this equation becomes Equation 8.

$$W = mg \tag{8}$$

where

W = weight.
m = mass.
g = acceleration due to gravity, 9.8 m/s^2.

The upward scale force, which exactly balances the downward force due to gravity, is defined as the "weight" of an object, and is the product of an object's mass m times g. Example E shows how to determine the weight of an object at the earth's surface.

EXAMPLE E

Given: An object has a mass of 25 kg.

Find: Weight of the object.

Solution:
$W = mg$
$= (25\ kg)(9.8\ m/s^2)$
$W = 245\ N$

As an object moves away from the surface of the earth, the value of g decreases, which causes the object's weight to decrease. If an object is placed upon another planet or moon, the acceleration due to gravity g can be calculated from the mass and radius of that planet or moon. This calculation would give a different acceleration due to gravity and, therefore, a different weight for the object. The same object upon the moon, for example, weighs about 1/5 as much as it does upon the earth.

SUMMARY

Each object in the universe exerts a gravitational force upon all others. This force is referred to as the "weight" of an object. The resistance of an object to a change in its motion is known as its mass. The influence of gravitational force upon the motion of bodies is important and can be calculated with use of the appropriate relationships.

MEASUREMENT OF WEIGHT AND MASS

MATERIALS

Triple-beam balance.
Spring balance (pound and Newton calibrated) with appropriate scale range.
Assorted weights.

PROCEDURE

In this experiment, the student will determine the weight and mass of several objects, using both a triple-beam balance and a spring balance.

1. Place first object on triple-beam balance, and adjust sliding weights until scale is balanced. Record mass of object (in grams) in Data Table. Notice that the beam balance determines mass and not weight.
2. Find weight of object by using Equation 8, converting mass to kilograms and multiplying by g (9.8 m/s^2). Record value in Data Table.
3. Find the weight of object in both pounds and Newtons, using spring balance, as in Figure 6. Record values in Data Table. Notice that the spring balance determines weight and not mass.
4. Calculate percent difference between weight of object found using the triple beam balance and its weight found using the spring balance, record this value.
5. Repeat Steps 1 through 4 for three additional objects. Record values in Data Table.

DATA TABLE

	Triple Beam Balance			Spring Balance			
	Measured mass in grams	Converted value in Newtons	Convrted value in pounds	Measured weight in Newtons	Measured weight in pounds	Percent Difference	
						N	lbs
Object 1							
Object 2							
Object 3							
Object 4							

SELF CHECK

1. With the help of Newton's "law of universal gravitation," explain how each object in the universe affects all other objects.

2. How do weight and mass differ? Is the mass of an object the same upon earth as upon any other planet? Can the same be said of weight?

3. Calculate the gravitational force exerted by the earth upon the moon. (The moon's mass is 7.34×10^{22} kg, and it is 3.86×10^{8} m from earth.)

4. Describe how force varies with distance in the inverse-square law.

5. Calculate the velocity of a free-falling object dropped from rest and falling for ten seconds. (Assume that no resistive forces act upon the object.)

6. Determine the mass of an object under the following conditions: The object has an acceleration of 5 m/s^2 when a force of 100 Newtons acts upon it and has an acceleration of 8 m/s^2 when a force of 160 Newtons acts upon it.

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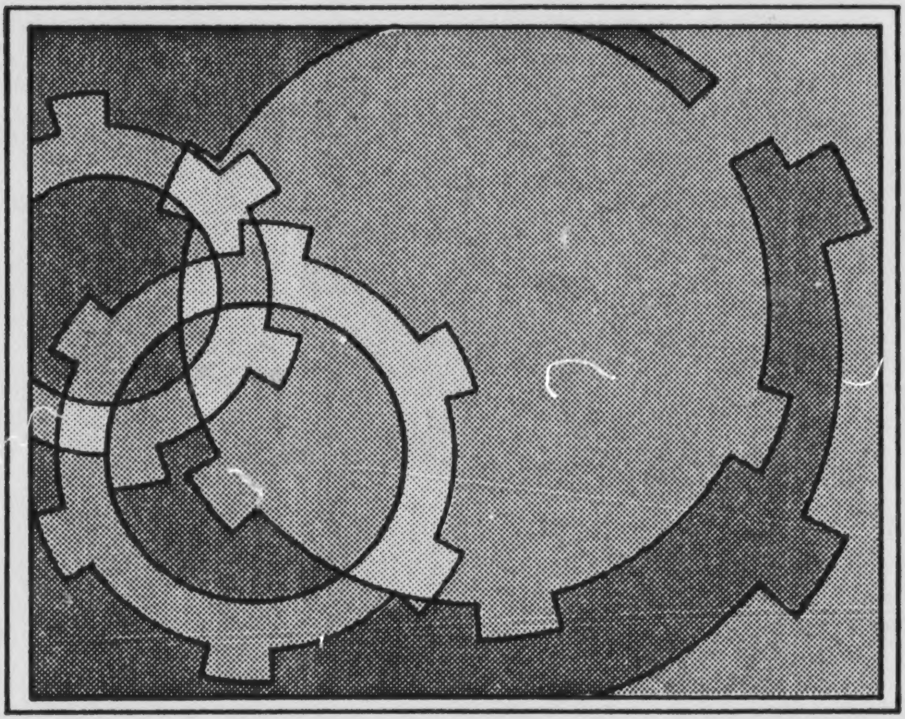
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 1 M 2

ADDITION OF MECHANICAL FORCES



MECHANICAL APPLICATION OF FORCE

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Force is the quality which changes, produces or stops the motion of a body; that is, it is the push-or-pull quality exerted upon an object. Two or more forces can be exerted at right angles, at other than right angles or along the same line (linear). The direction and magnitude that results when two or more forces are exerted upon an object is called the "resultant." This module presents graphical and mathematical methods for finding resultants. The methods explained in this module are valid for any vector qualities. Although most of the examples given deal with forces, the same resolution and composition procedures can be used for any other type of vector qualities, such as velocities, accelerations and momenta. Resultants are extremely important in a wide range of technical applications. In the laboratory, the student will determine several resultants experimentally, graphically and mathematically.

PREREQUISITES

Student should have completed Concept Module 2-0, "Force," and should understand the definitions of the terms "sine," "cosine" and "tangent" and be able to calculate values of these functions, using trig tables, slide rule or electronic calculator.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:

  Contact forces.
  Action-at-a-distance forces.
  Concurrent forces.
  Coplanar forces.
  Resultant.

- Determine the resultant of concurrent forces, graphically, mathematically or experimentally for the follwing four cases:

Two forces at right angles.
Two forces not at right angles.
Both forces in the same line.
More than two coplanar forces.

- Given the appropriate equipment, determine the resultants of forces in several directions.

DISCUSSION

Forces are exerted by a hammer hitting a nail, a brace holding up a load, a stretched spring on the bodies to which it is attached, a tractor pulling a plow, a rope lifting a bale of hay. In all these examples, force is in contact with the body upon which it is exerted, and is known as "contact" force. There are also forces that act without making contact. These forces are called "action-at-a-distance" forces; for example, the gravitational attraction, or weight, exerted upon a body by the earth. All known forces are in one of these two classes.

If a number of forces are applied at a common point, they are "concurrent." If all of the forces lie in the same plane, they are called "coplanar." If all forces acting upon a system are concurrent, to find a simple force with the proper magnitude and direction that produces the same effect is possible. This force, called the "resultant," will be determined for four different cases in this module.

The four cases of concurrent, coplanar forces considered in this module are studies in sequence. Graphical and mathematical determinations of resultants will be compared with experimental results.

TWO FORCES AT RIGHT ANGLES

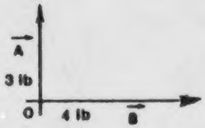


FIG. 1

Two forces $\vec{A}$ and $\vec{B}$ are applied simultaneously at the point O, as shown in Figure 1. Arrows ordinarily are used over the letters to indicate that A and B are vector quantities.

GRAPHICAL SOLUTION

There are two methods of determining the resultant force graphically. As discussed in Concept Module -0, resultant force is that force that replaces the others, in this case A and B.

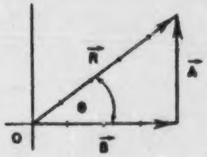


FIG. 2

1. Triangle method. Force $\vec{A}$ is laid off with its tail at the head of force $\vec{B}$, as in Figure 2, with some convenient scale.

   The closing side of the triangle, $\vec{R}$, represents the resultant. Magnitude of the resultant, R, can be measured with an appropriate scale, and the angle can be measured with a protractor. The same resultant would be obtained if force $\vec{B}$ were added to force $\vec{A}$, as in Figure 3, which indicates that the order of adding forces is not important, since the same resultant will always be obtained.

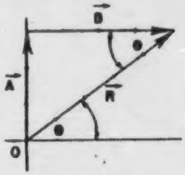


FIG. 3

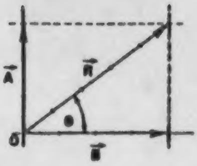


FIG. 4

2. Parallelogram method. To find the resultant force using this method, the given forces $\vec{A}$ and $\vec{B}$ are laid off to scale horizontal and vertical construction lines are drawn parallel to $\vec{A}$ and $\vec{B}$ as in Figure 4. The resultant, $\vec{R}$, will be the diagonal of the parallelogram, as shown. As in the triangle method, the magnitude of $\vec{R}$ and the angle θ can be measured with a scale and a protractor, respectively. In these examples, forces $\vec{A}$ and $\vec{B}$ were chosen to be 3 lb and 4 lb, respectively. In all the graphical solutions, the resultant R should be 5 pounds, and the angle θ should be 37°.

MATHEMATICAL SOLUTION

Because the forces $\vec{A}$ and $\vec{B}$ act at right angles to each other and can be added as shown in Figure 2, the resultant can be computed from the right triangle formed by $\vec{A}$, $\vec{B}$ and $\vec{R}$, as indicated by Equations 1 and 2.

$$R^2 = A^2 + B^2 \qquad (1)$$

where

R = resultant.
A = vertical force.
B = horizontal force.

$$R = \sqrt{A^2 + B^2} \qquad (2)$$

Example A shows the use of Equation 2 in solving a problem.

EXAMPLE A: TWO FORCES AT RIGHT ANGLES -MATHMATICAL SOLUTION

Given: A vertical force A of 3 lb and a horizontal force B of 4 lb, as shown in Figure 1.

Find:
(a) The resultant R.
(b) The angle θ.

Solution: (a) The resultant R can be found by Equation 2 as follows:

$$R = \sqrt{A^2 + B^2}$$

Because A = 3 lb and B = 4 lb, Equation 2 becomes:

$$R = \sqrt{(3\ \text{lb})^2 + (4\ \text{lb})^2} = \sqrt{9\ \text{lb}^2 + 16\ \text{lb}^2}$$

$$R = \sqrt{25\ \text{lb}^2}$$
$$R = 5\ \text{lb}$$

(b) The angle θ can be computed from either its sine, cosine or tangent, as follows:

$$\sin\theta = \frac{A}{R} = \frac{3\ \text{lb}}{5\ \text{lb}} = 0.6$$
$$\cos\theta = \frac{B}{R} = \frac{4\ \text{lb}}{5\ \text{lb}} = 0.8$$
$$\tan\theta = \frac{A}{B} = \frac{3\ \text{lb}}{4\ \text{lb}} = 0.75$$

The angle θ for each of these values is shown from angle tables to be as follows:

$$\theta = 37^\circ$$

Notice that these results are in agreement with those obtained graphically. In summary, a force of 5 lb acting at an angle of 37° with the horizontal will produce the same effect as the two forces of 3 lb vertically and 4 lb horizontally. The resultant is not the arithmetical sum of 3 lb and 4 lb. In other words, the two forces are not equivalent to a single force of 7 lb.

TWO FORCES NOT AT RIGHT ANGLES

$\vec{C}$ and $\vec{D}$ in figure 5 represents two forces at an angle α whose resultant is to be determined. The forces are not at right angles to each other.

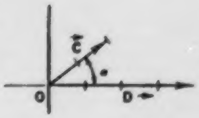
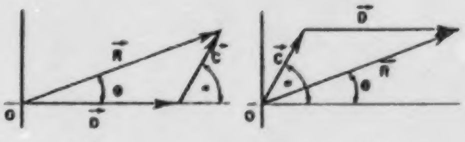


FIG. 5

GRAPHICAL SOLUTION

The two graphical methods used in the previous case will be applied here, as the techniques are very similar.

1. Triangle method. Choose a convenient scale and lay off force $\vec{C}$ with its tail at the head of force $\vec{D}$ as in Figure 6a. The force $\vec{D}$ can also be drawn with its tail at the head of $\vec{C}$ as in Figure 6b with the same results. The resultant $\vec{R}$ and the angle θ can then be measured with a scale and protractor respectively.



(a) (b)

FIG. 6

2. Parallelogram method. Draw dashed construction lines parallel to $\vec{C}$ and $\vec{D}$, as in Figure 7. The resultant is the diagonal of the parallelogram as shown in this figure. The resultant $\vec{R}$ and the angle θ can be determined by using a

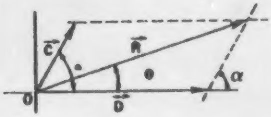


FIG. 7

scale and a protractor, respectively. In these examples, forces $\vec{C}$ and $\vec{D}$ are 2 lb and 4 lb, respectively, and are applied at 45° to each other. In all of the graphical solutions, the resultant R is found as 5.60 lb and the angle θ, 14.6°.

MATHMATICAL SOLUTION

For two forces not at right angles, there are two mathematical approaches that can be used.

1. Method law of sines and cosines. The law of sines and cosines for the oblique triangle in Figure 8 are indicated by Equations 3, 4 and 5.

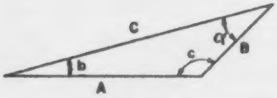


FIG. 8

$$C^2 = A^2 + B^2 - 2AB \cos C \quad \text{law of cosine} \tag{3}$$

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \quad \text{law of sines} \tag{4}$$

The law of cosines to the triangle that was formed in Figure 6a yields the results indicated in Equation 5.

$$R^2 = C^2 + D^2 - 2CD \cos (180° - \alpha) \tag{5}$$

These equations are used to solve a problem in Example B.

EXAMPLE B: TWO FORCES NOT AT RIGHT ANGLES - MATHMATICAL SOLUTION

Given: The following values for use with Figure 6a: $C = 2$ lb, $D = 4$ lb and $\alpha = 45°$.

Find: (a) The resultant R^2.
(b) The angle θ.

Solution: (a) The resultant R^2 can be found by Equation 5 as follows:

$$\begin{aligned} R^2 &= C^2 + D^2 - 2CD \cos (180° - \alpha) \\ &= (2)^2 + (4)^2 - 2(2)(4) \cos (180° - 45°) \\ &= 4 + 16 - 16 \cos 135° \end{aligned}$$

$$R^2 = 20 - 16\ (-0.707)$$
$$R^2 = 20 + 11.3 = 31.3$$
$$R = \sqrt{31.3} = 5.59\ \text{lb}$$

(b) The angle θ in Figure 6a can be computed from the law of sines as follows:

$$\frac{C}{\sin\theta} = \frac{R}{\sin\ (180^\circ - \alpha)}$$
$$\sin\theta = \frac{C}{R}\sin\ (180^\circ - \alpha)$$
$$\sin\quad = \frac{2}{5.59}\sin\ (180^\circ - 45^\circ)$$
$$\sin\theta = 0.2530$$
$$\theta \approx 14.6$$

2. Rectangular resolution method. The method of law of sines and cosines for finding the resultant is often awkward and, when applied to more than two forces, becomes very difficult to manage. The rectangular resolution method to be explained here is easier to handle, and can be used with any number of forces. Indeed, this method is probably the best for computational purposes to determine the resultant force in any situation; it can be outlined in 3 steps:

   a. Resolve all forces into their rectangular components along the X and Y axis.
   b. Find the algebraic sum of all X and Y components. The problem now has been reduced to one net force along X and one net force along Y.
   c. Combine these two net forces to find the resultant, as for a right triangle explained in Figure 2 or 3.

   As an example, the resultant of the two forces C and D used previously are computed in Example C.

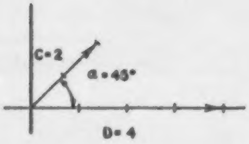
EXAMPLE C: TWO FORCES NOT AT RIGHT ANGLES - THE RECTANGULAR RESOLUTION METHOD

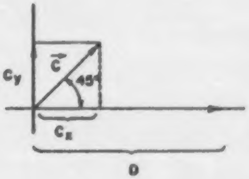
Given: The following values (same as in Example B) for use with first two sketches below: C = 2 lb, D = 4 lb and α = 45°.

Find: (a) The resultant R.
(b) The angle θ.

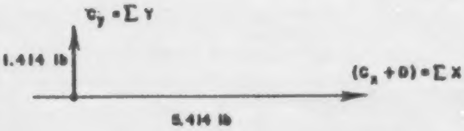
Solution: (a) The resultant R can be found with the rectangular resolution method as indicated below:

Since force D already lies on the X axis (first adjoining sketch), resolve force C into its X and Y components (second adjoining sketch). X components directed to the right are usually considered positive and those to the left negative. Similary, Y components directed upward are positive, and those downward are negative.

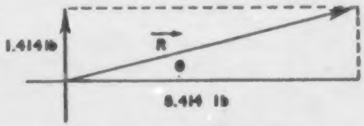




The X components of the 2-lb $\vec{C}$ force is 2 cos 45° = 2 (0.707) = 1.414 lb. This component added to the 4-lb $\vec{D}$ force already in the positive X direction yields a net force of 1.414 lb + 4 lb = 4.414 lb to the right. The Y component of the 2-lb $\vec{C}$ force is cos 45° = 2 (0.707) = 1.414 lb upward. Because the 4-lb force has no vertical component, the algebraic sum of all the Y components is simply 1.414 lb. These resultant X and Y components are shown in adjoining sketch.



The sumbols ΣX and ΣY stand for the sum of the X and Y components, respectively. The resultant now can be calculated as before, as shown in adjoining sketch.



$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$$

$$= \sqrt{5.414^2 + 1.414^2} = \sqrt{29.31 + 2.000}$$

$$= \sqrt{31.31}$$

$$R = 5.60 \text{ lb}$$

This value agrees with the result previously calculated.

(b) The angle θ can be computed from any of the trigonometric functions, but only one will be used here.

$$\tan\theta = \frac{1.414\ \text{lb}}{5.414\ \text{lb}}$$

$$= 0.2611$$

$$\theta = 14.6°$$

θ also agrees with the result previously calculated.

BOTH FORCES IN THE SAME LINE

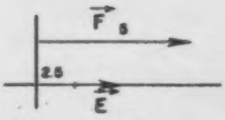
When both forces lie in the same straight line, the triangle shown in Figure 6a can be represented as flattening out into a straight line. The adding and subtracting of numbered line segments was used in Example C to obtain resultant X and Y components. All these forces are displaced slightly in sketches of Example D in order that they may be observed. When both forces are in the same line, the resultant R can be found as shown in Example D.

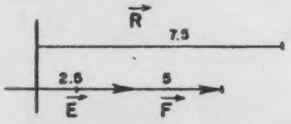
EXAMPLE D: BOTH FORCES IN THE SAME LINE

Given: Adjoining sketches and the following values: Force $\vec{E} = 2.5$ lb, and Force $\vec{F} = 5$ lb.

Find: Resultant $\vec{R}$.

Solution: $\vec{R}$ is therefore simply 2.5 lb + 5 lb = 7.5 lb, in the positive X direction. If the forces are in the same line, but are oppositely directed as in Figure 9, the results are obtained in a similar fashion. In this case, the resultant $\vec{R}$ is 2.5 lb - 5 lb = -2.5 lb. The minus sign simply indicates that the resultant is directed in the negative X direction.





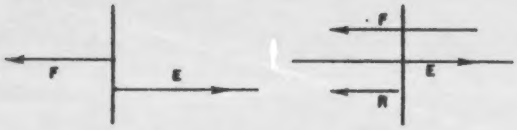
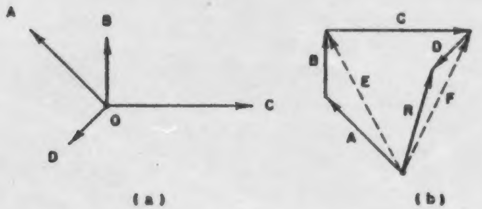


FIG. 9

MORE THAN TWO FORCES

When the resultant of two or more forces is being computed graphically or mathematically, methods are essentially the same as in Example D. This section shows how these methods can be extended to handle several forces acting together.

GRAPHICAL SOLUTION - POLYGON METHOD



(a) (b)

FIG. 10

As more than two forces are being resolved, the triangle method in reality becomes the better-known polygon method. This process, illustrated in Figure 10a, shows four forces $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ acting concurrently at a point O. In Figure 10b, forces $\vec{A}$ and $\vec{B}$ are combined graphically, giving the resultant $\vec{E}$. The resultant force $\vec{E}$ is then combined graphically with force $\vec{C}$, giving the resultant $\vec{F}$. Finally, force $\vec{F}$ is combined graphically with force $\vec{D}$, giving the final resultant $\vec{R}$. This process could be simplified, because forces $\vec{E}$ and $\vec{F}$ need not have been drawn. The given forces are drawn in succession, with the tail of each at the head of the force preceding it. The resultant is then the force that completes this polygon, and is drawn from the tail of the first force to the head of the last. To illustrate that the order of adding the forces is unimportant, they are added in a different sequence in Figure 11. The same resultant would occur for any other sequence. For an interesting exercise, this claim should be verified. When the resultant has been obtained in this manner, its magnitude R and its direction θ can be determined with the aid of a scale and a protractor.

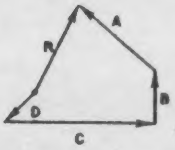


FIG. 11

MATHMATICAL METHOD - RECTANGULAR RESOLUTION

Because the "law of sine and cosine method" requires computations involving oblique triangles and is awkward, only the "method of rectangular resolution" will be shown. The resultant of the four forces shown in Figure 10a will be computed in Example E. The magnitude and angles of these forces are indicated in Figure 12a.

Rectanglular resolution is demonstrated in Example E.

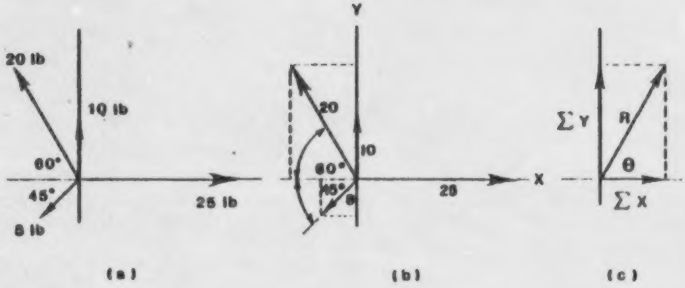


FIG. 12

EXAMPLE E: RECTANGULAR RESOLUTION OF MORE THAN TWO FORCES

Given: The below values for use with Figure 12b, which resolves each force into X and Y components.

Force Vector	X-component	Y-component
10 lb	0	+ 10 lb
25 lb	+ 25 lb	0
20 lb	-[illegible]0 cos 60 = - 10 lb	+20 sin 60 = 17.32 lb
8 lb	-8 cos 45 = -5.66 lb	-8 sin 45 = -5.66 lb

Find: (a) Algebraic sum of X and Y components.
(b) The resultant R.

Solution: (a) The algebraic sum of the X component forces is 9.34 lb directed toward the right, and the algebraic sum of the Y component forces is 21.66 lb directed upward.

(b) Magnitude and direction of the resultant can be determined as before for a right triangle as shown in Figure 12c.

$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2} = \sqrt{(9.34)^2 = (21.66)^2} = \sqrt{556.39}$$

$$R = 23.6 \text{ lb}$$

$$\tan \theta = \frac{\Sigma Y}{\Sigma X}$$

$$= \frac{21.7 \text{ lb}}{9.34 \text{ lb}}$$

$$\tan \theta = 2.32.$$

$$\theta = 66.7°$$

These results are accurate. The results obtained graphically will agree well with these results if the drawings and measurements with scale and protractor are carefully done.

APPLICATION TO THE VELOCITY VECTOR

As previously stated, vector methods outlined in this module are not restricted to force vectors, but apply as well to any other vector quantities. Following is a typical example in which the vector is quantity of interest in velocity.

Suppose a pilot flies in a NE direction (45° north of east) at an indicated airspeed of 160 mi/h. During the flight, a wind is blowing at 50 mi/h due south. What is the resultant velocity of the airplane with respect to the ground?

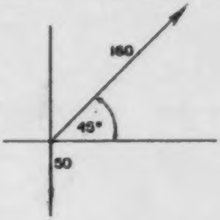
Vectorial solution to a velocity problem is shown in Example F.

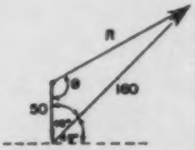
EXAMPLE F: VECTOR APPLIED TO VELOCITY

Given: A pilot flies in a northeastern direction (45° north of east), at an indicated airpseed of 160 mi/h. During the flight, a wind is blowing at 50 mi/h due south.

Find: The resultant velocity of the airplane with respect to the ground.

Solution: The first adjoining sketch shows the two velocities in the directions specified. The second is the vector triangle that must be solved for the resultant velocity R. Use the cosine law for the magnitude of R:





$$R^2 = 50^2 + 160^2 - 2(50)(160) \cos 45^\circ$$

$$R^2 = 16786$$

$$R = 129.6 \text{ mi/h}$$

Use the sine law for the direction of R:

$$\frac{\sin \theta}{\sin 45} = \frac{160}{129.6}$$

$$\sin \theta = \frac{160}{129.6} (\sin 45^\circ)$$

$$= 0.8730$$

$$\theta = 60.8^\circ$$

The aircraft would be traveling at 129.6 mi/h in a direction 60.8° north of east. Vectors are used widely in navigation.

SUMMARY

Two or more forces (qualities of push or pull upon an object) can be exerted at right angles, at other than right angles or along the same line. The "resultant" is the resulting direction and magnitude of force, and can be determined mathematically or graphically. Many systems in technology and industry rely upon vectorial solutions of force problems and of numerous other problems.

DETERMINING RESULTANTS OF FORCES

MATERIALS

Scale.
Protractor.
Force table and accessories.

PROCEDURE

1. Level force table; attach weights; position pulleys to arrange force table (Figure 13) for Problem I as shown in Data Table.

2. Experimentally determine angle and weight required to balance given forces in order that small ring remains stationary when pin is removed. Because system is balanced, resultant of forces is equal in magnitude to this weight, but is oppositely directed. Record both magnitude and direction of resultant in appropriate boxes in Data Table for Problem I.

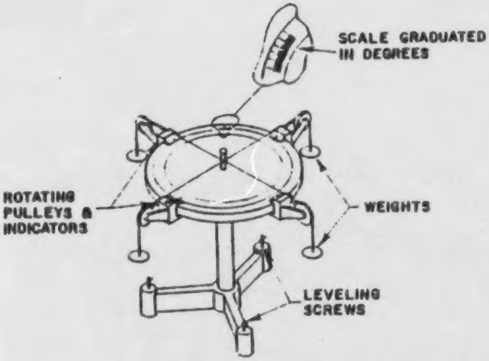


FIG. 13

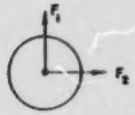
3. Graphically determine magnitude and direction of resultant; record in appropriate boxes.

4. Mathematically determine magnitude and direction of resultant; record in appropriate boxes.

5. Repeat steps 1, 2, 3 and 4 for Problems II, III and IV as illustrated in Data Table.

6. Do the experimental, graphical and/or mathematical resultants for any one of the four problems differ? If so, why?

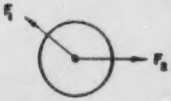
DATA TABLE



Problem I

	Gram Weights	Angle
F_1	150	90°
F_2	300	0°

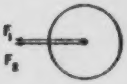
	Resultant
Experimental	
Graphical	
Mathematical	



Problem II

	Gram Weights	Angle
F_1	100	120°
F_2	350	0°

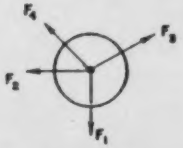
	Resultant
Experimental	
Graphical	
Mathematical	



Problem III

	Gram Weights	Angle
F_1	200	180°
F_2	150	180°

	Resultant
Experimental	
Graphical	
Mathematical	



Problem IV

	Gram Weights	Angle
F_1	200	270°
F_2	400	180°
F_3	50	45°
F_4	150	150°

	Resultant
Experimental	
Graphical	
Mathematical	

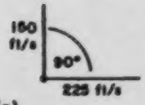
SELF CHECK

1. Define:

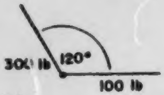
Contact forces.
Action-at-a-distance forces.
Concurrent forces.
Coplanar forces.

Resultant.

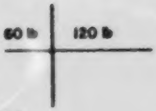
2. Determine the resultant of concurrent forces, graphically and experimentally for the following:


160 ft/s
90°
225 ft/s

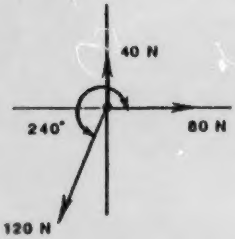
(a)


300 lb
120°
100 lb

(b)



(c)


40 N
240°
80 N
120 N

(d)

(e) 25 ft at 30°.
32 ft at 120°.
15 ft at 300°.

(f) 4.2 N at 37°.
6.8 N at 58°.
1.3 N at 198°.

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Sears, F.W. and Zemansky, M.W. 3rd ed. College Physics. Reading, MA: Addison-Wesley Publishing Company, Inc., 1960.

Williams, J.E.; Metcalfe, H. Clark; Trinklein, Frederick E.; and Lefler, Ralph W. Modern Physics. New York: Holt, Rinehart and Winston, 1968.

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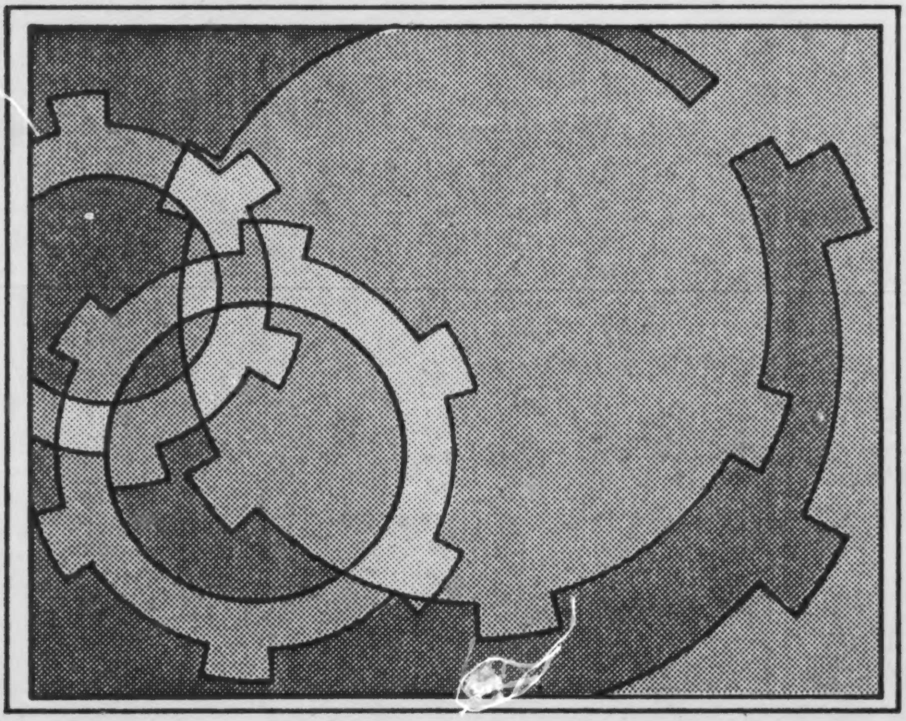
UNIFIED TECHNICAL CONCEPTS

MODULE 1 M 3

ADDITION OF TORQUES



MECHANICAL APPLICATION OF FORCE

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

Torque (moment) is the force-like quality in the mechanical rotational system. The twisting action of torque results when all forces acting upon a body do not act through a single point; that is, force is applied off the center of mass, causing rotation. Normally, the word "torque" is applied to this rotational movement, and the term "moment" is employed to describe the product of the force and its moment arm (perpendicular distance from axis to point at which force is applied). Rotational equilibrium is the state at which the forces applied will not cause rotation about an axis.

In this module, the student will use the concept of torque to solve problems involving torque and equilibrium. In the laboratory, the student will determine the mass of an object from the forces applied to it, and will determine the position required to balance it, or bring it to equilibrium.

PREREQUISITES

Student should have completed Concept Module I-0, "Force," and Application Module IM2, "Addition of Mechanical Forces."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:

    Torque.
    Moment of a force.
    Moment arm.
    Center of gravity.

- Describe in words and with equations the two conditions necessary for equilibrium of a body.

- Given the forces acting on a body and the moment arm of each force, calculate the force and moment arm necessary to establish equilibium.
- Given the appropriate equipment, experimentally verify the solution to equilibrium problems and compare the calculated and experimental results.

DISCUSSION

The first condition for equilibrium is given by Equation 1.

$$\Sigma\vec{F} = 0 \qquad (1)$$

where

$\Sigma\vec{F}$ = the vector sum of forces acting upon a body.

When this condition is satisfied, the body is in "translational equilibium"; that is, the forces will not cause it to translate as a whole along a fixed direction. Figure 1 shows a mechanical system for which this condition is met. This system is not, however, in complete equilibrium. The two forces have equal magnitudes and opposite directions, but they do not act through the same point. As a result, the two forces cause a rotation of the body. In this figure, point P is an axis perpendicular to the paper about which the body is free to rotate. The torque (or moment) of a force is given by Equation 2. The word "torque" normally is used when movement results, and the word "moment" (or torque) of a force about an axis is the product of the force and its lever arm.

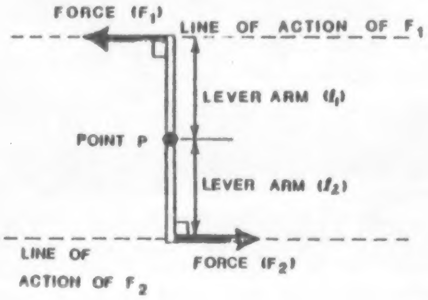


FIG. 1

$$\tau = F\ell \qquad (2)$$

where

τ = torque.
F = force.
ℓ = lever arm.

The lever, or moment, arm is the perpendicular distance from the line of action of the force to the axis, or pivot point. Since a clockwise rotation is positive, the total torque for this system is indicated by Equation 3, in which both torques cause counterclockwise rotations.

$$\Sigma\tau = (-F_1\ell_1) + (-F_2\ell_2) \qquad (3)$$

where

$\Sigma\tau$ = total torque.
F_1 = force 1.
F_2 = force 2.
ℓ_1 = lever arm 1.
ℓ_2 = lever arm 2.

The second condition of equilibrium is given in Equation 4, in which the sum of the moments on a body about any axis is zero.

$$\Sigma\tau = 0 \qquad (4)$$

When this condition is satisfied, the body is in "rotational equilibrium"; that is, the forces will not cause it to rotate about any fixed axis. When both conditions for equilibium (Equation 1 and Equation 4) are met, the forces will not cause either translation or rotation of the body.

Figure 2 shows a system which meets both conditions for equilibrium. Equilibrium in this situation can be verified by the use of Equations 1, 2 and 4 in Example A.

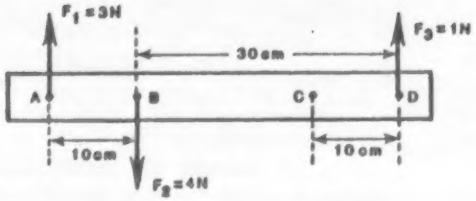


FIG. 2

EXAMPLE A: EQUILIBRIUM SYSTEM CALCULATION

Given: The mechanical system in Figure 2.

Find: If the system is in eqiulibium.

Solution:

1. $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ (Define up to be positive.)
   $= 3\ N + (-4\ N) + 1\ N$
   $\Sigma\vec{F} = 0.$ (First condition for equilibium is met.)
2. $\Sigma\tau = F_1\ell_1 + F_2\ell_2 + F_3\ell_3$ (Define clockwise to be positive.)
   (a) Choose point A as the pivot point - the axis.
   $\Sigma\tau = (3\ N)(0\ M) + (+4\ N)(0.1\ M) + (-1\ N)(0.4\ M)$
   $= +0.4\ N\cdot m - 0.4\ N\cdot m$
   $\Sigma\tau = 0$ (Second condition for equilibium is satisfied.)

Notice that F_1 causes no rotation about point A, F_2 gives a positive torque and F_3 gives a negative torque.

(b) Choose point D as the pivot point.

$\Sigma\tau = (3\ N)(0.4\ m) - (4\ N)(0.3\ m) + (1\ N)(0\ m)$

$(1.2\ N{\cdot}m - 1.2\ N{\cdot}m$

$\Sigma\tau = 0$

(c) Choose point C as the pivot point.

$\Sigma\tau = (3\ N)(0.3\ m) - (4\ N)(0.2\ m) - (1\ N)(0.1\ m)$

$= 0.9\ N{\cdot}m - 0.8\ N{\cdot}m - 0.1\ N{\cdot}m$

$\Sigma\tau = 0$

Notice that F_1 causes a clockwise rotation about point C and gives a positive torque. F_2 and F_3 both cause counterclockwise rotations and give negative torques.

Example A shows that if a body is in rotational equilibrium about some axis, it is in equilibrium about any axis; however, it is only necessary to calculate torques about one point. In systems in which a fixed axis of rotation is defined, that axis is usually chosen. (Any convenient point may be used in testing for equilibrium of torques.) In systems involving the action of gravitational forces, the center of gravity may be chosen.

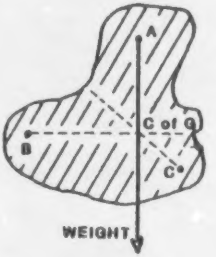
The center of gravity (C of G) of an object generally is defined as that point at which the entire weight of an object can be considered to be concentrated. An object will balance if supported or hung from its center of gravity. Example B shows how to find the C of G of nonuniform density body of irregular shape.

EXAMPLE B: CALCULATION OF IRREGULAR BODY C OF G

Given: The irregular object shown in the figure.

Find: C of G.

Solution: (a) Suspend the object near the periphery point A; drop a weighted line from A and mark its location. (b) Suspend the object near the periphery, say point B; drop a weighted line from B, and mark where it crosses line A. (c) Suspend the object near the periphery, say point C; drop a weighted line from C, and it should cross the intersection of lines A and B at the C of G.

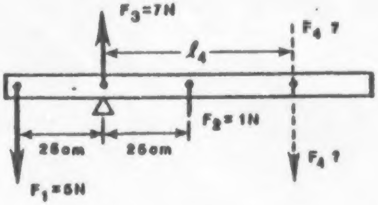


Example C is another problem using Equations 1, 2 and 4 in the solution. In this example, "down" and "counterclockwise" were chosen as positive. This was done to stress the point that positive and negative directions may be defined in any way, as long as they are consistent within any single problem.

EXAMPLE C: EQUILIBRIUM BEAM CALCULATION

Given: The system shown in figure.

Find: F_4 and ℓ_4 that will cause the system to be in equilibrium about Δ.



Solution: (a) $\Sigma\vec{F} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$\vec{F}_4 = -\vec{F}_1 - \vec{F}_2 - \vec{F}_3$ (Define down to be positive.)

$= -(5\text{ N}) - (1\text{ N}) - (-7\text{ N})$

$= -5\text{ N} - 1\text{ N} + 7\text{ N}$

$\vec{F}_4 = 1\text{ N}$. (F_4 is 1 N downward.)

(b) $\Sigma\tau = 0 = F_1\ell_1 + F_2\ell_2 + F_3\ell_3 + F_4\ell_4$. [Define ccw (counterclockwise) as positive and the fulcrum as reference point.]

$$0 = (5\text{ N})(25\text{ cm}) - (1\text{ N})(25\text{ cm}) + (7\text{ N})(0) + (1\text{ N})(\ell_4)$$

$$= 125\text{ N}\cdot\text{cm} - 25\text{ N}\cdot\text{cm} + 0 - 1\text{ N} \times \ell_4$$

$$0 = 100\text{ N}\cdot\text{cm} - 1\text{ N} \times \ell_4$$

$$-100\text{ N}\cdot\text{cm} = 1\text{ N} \times \ell_4$$

$$\frac{-100\text{ N}\cdot\text{cm}}{-1\text{ N}} = \ell_4$$

$$100\text{ cm} = \ell_4$$

$$1\text{ m} = \ell_4$$

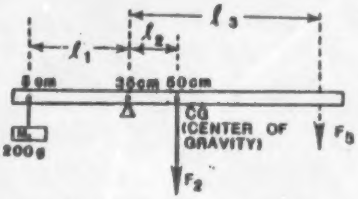
Since ℓ_4 equals one meter, F_4 is one meter to the right of the fulcrum.

Example D shows the solution to an additional equilibrium problem. Since mass units (grams) are given rather than force units (newtons), the acceleration due to gravity (g) is used in the solution even though it divides out.

EXAMPLE D: EQUILIBRIUM BEAM CALCULATION

Given: The system shown in figure.

Find: Where a 100 g weight should be placed to achieve equilibrium.



M_2 = MASS OF METERSTICK = 50g

CG = CENTER OF GRAVITY

Solution: $m_1 = 200\text{ g},\ m_2 = 50\text{ g},\ m_3 = 100\text{ g}$

$\ell_1 = 30\text{ cm},\ \ell_2 = 15\text{ cm},\ \ell_3 = ?$ (Choose fulcrum as reference point.)

$\Sigma\tau = 0 = F_1\ell_1 + F_2\ell_2 + F_3\ell_3 + F_4\ell_4$

$0 = -m_1g\ell_1 + m_2g\ell_2 + m_3\ell_3g + F_4\ell_4$ (F_4 is upward force at fulcrum since ℓ_4 is zero; F_4 need not be calculated.)

$= g\ (-m_1\ell_1 + m_2\ell_2 + m_3\ell_3)$ (Since g is in every term, it can be divided out of the equation.)

$= -\ (200\text{ g})(30\text{ cm}) + (50\text{ g})(15\text{ cm}) + (100\text{ g})(\ell_3)$

$= -6000\text{ g}\cdot\text{cm} + 750\text{ g}\cdot\text{cm} + 100\text{ g}\ \ell_3$

$0 = -5250\text{ g}\cdot\text{cm} + 100\text{ g}\ \ell_3$

$100\text{ g}\ \ell_3 = 5250\text{ g}\cdot\text{cm}$

$$\ell_3 = \frac{5250\text{ g}\cdot\text{cm}}{100\text{ g}}$$

$\ell_3 = 52.5\text{ cm}$

The 100 g weight should be placed 52.5 cm to the right of the reference point (fulcrum) or at the 87.5 cm mark.

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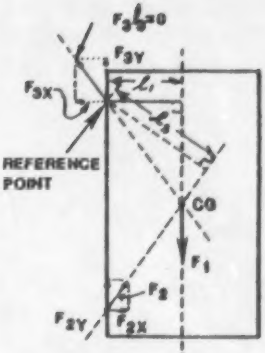

FIG. 3

In Example D, all forces have been applied in a "single" direction; that is, either up or down. In many "real" problems this is not the case. Figure 3 shows the forces F_1, F_2 and F_3, acting on a door, together with their X and Y components, and their lever arms.

The reference point chosen in Figure 3 is the upper hinge, and the equilibrium conditions are given by Equations 5, 6 and 7:

$$\Sigma F_X = 0 = F_{2_X} - F_{3_X} \qquad (5)$$

where
X = equilibrium of X-component forces.

$$\Sigma F_Y = 0 = F_{2_Y} + F_{3_Y} - F_1 \qquad (6)$$

where
Y = equilibrium of Y-component forces.

$$\Sigma \tau = 0 = F_1\ell_1 - F_2\ell_2 + F_3\ell_3 \qquad (7)$$

Equilibium of torques is shown by Equation 7.

SUMMARY

Torque is the twisting, force-like quality in the mechanical rotational system. Torque is produced when all forces acting upon a body do not act through a single point, but act off the axis, causing rotation. The moment, or lever, arm is the point at which force is applied perpendicularly away from the axis. Rotational equilibrium is the state at which the forces applied will not cause rotation.

MASS OF METER STICK

MATERIALS

Meter stick (weight clamps optional).
String.
Set of metric weights.
Stand or mount on which system can be hung.

PROCEDURES

1. Suspend a 100-g weight from 10-cm point on meter stick with a small piece of string.
2. Tie a loop in a short length of string, and slip it over free end of meter stick. Exert an upward force on string. This string is shown as "F" in Figure 4. Move the loop of string to the position which balances the meter stick while hanging freely.
3. Record all necessary data in Data Table, and calculate mass of meter stick.

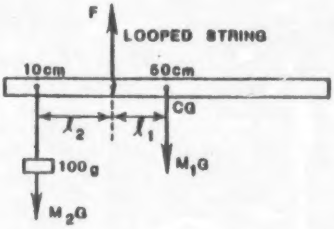

FIG. 4

MASS NECESSARY TO BALANCE METER STICK

MATERIALS

Meter stick (weight clamps optional).
String.
Set of metric weights.
Stand or mount on which system can be hung.

PROCEDURE

1. Hang weights from meter stick, as shown in Figure 5, and place string supporting meter stick at the 65-cm mark.

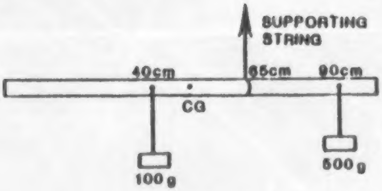

FIG. 5

2. Calculate mass placed at the 15-cm mark necessary to balance meter stick, and record in Data Table.

3. Hang weights at 15-cm mark on stick until it is balanced. Record this mass as that necessary to balance experimentally this system.

4. What is the relationship between masses and moment levers?

POSITION OF GIVEN MASS TO BALANCE METER STICK

MATERIALS

Meter stick (weight clamps optional).
String.
Set of metric weights.
Stand or mount on which system can be hung.

PROCEDURE

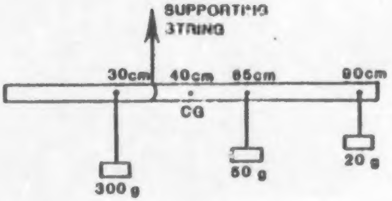

FIG. 6

1. Assemble experimental setup as shown in Figure 6.

2. Calculate location of 200-g weight that balances the stick. Record in Data Table.

3. Hang a 200-g weight from stick and move it until a balance is achieved experimentally. Record this position in Data Table.

4. In what state are torque forces?

DATA TABLE

Procedure 1

Mass of weight : $m_2 = 100$ g.
Position of looped string on meter stick that causes blance _________ cm.
Distance from looped string to center of gravity of stick: $\ell_1 =$ _________ cm.
Distance from looped string to weight: $\ell_2 =$ _________ cm.
Since $\Sigma\tau = 0$, $m_1 g \ell_1 = m_2 g \ell_2$,

$$\text{Mass of meter stick: } m_1 = \frac{m_2 \ell_2}{\ell_1}$$

= (_______ g)(_______ cm) ÷ (_______ cm)
= _______ g.

Procedure 2

Mass calculated to balance meter stick: _______ g.
Mass that balances stick experimentally: _______ g.

Procedure 3

Location calculated to balance meter stick: _______ g.
Location that balances stick experimentally: _______ g.

SELF CHECK

1. Identify each of the following:
   a. The point at which the entire weight of an object can be considered to be concentrated.
   b. The perpendicular distance from the line of action of a force to the pivot point or axis of rotation.
   c. The product of a force and its lever arm when no motion occurs.
   d. The product of a force and its lever arm when rotational motion results.
2. Describe the two conditions necessary for equilibrium of a body.
3. A meter stick with a mass of 162 g is suspended by a string from the 40-cm mark. The following weights are hung from

the stick at the specified locations. Find the missing quantity for equilibrium in each case.

a. $m_1 = 100$ g $m_2 = 50$ g
$X_1 = 30$ cm $X_2 =$ ________ cm
b. $m_1 = 500$ g $m_2 =$ ________ g
$X_1 = 40$ cm $x_2 = 10$ cm
c. $m_1 = 100$ g $m_2 = 200$ g $m_3 = 50$ g.
$X_1 = 10$ cm $X_2 = 50$ cm $X_3 =$ ________ cm

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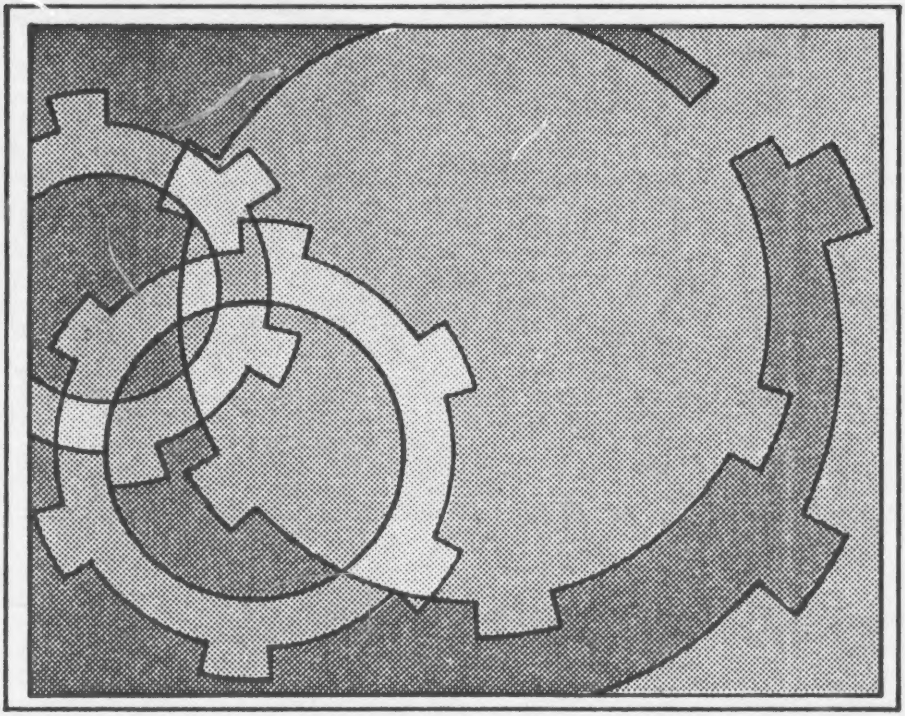
UNIFIED TECHNICAL CONCEPTS

MODULE 1 M 4

MECHANICAL EQUILIBRIUM IN STRUCTURES



MECHANICAL APPLICATION OF FORCE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

In preceding modules, relationships of forces and motions were discussed. Force always tends to cause a change in motion of a body, and torques will cause a body to rotate about a selected point. Bridges, buildings, dams and houses are designed to withstand forces and torques of wind, earthquakes, internal weight and motion of water. Because forces and torques cause motion, it is the problem of the designer to distribute these forces and stresses in individual members that make up the structure so that they are all in equilibrium. This module considers application of forces and torques to everyday common structures to ensure that their design is within the safe limits of loading.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force," and Application Modules IM2 "Addition of Mechanical Forces" and IM3 "Addition of Torques."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Static load.
  - Perfect structure.
  - Strut.
  - Tie.
  - Frame structure.
- Draw a simple loading diagram for a structural system, identifying load location and direction.
- Describe the two conditions of equilibrium and apply them to a perfect structure.

- Calculate forces in a perfectly framed structure using methods of vectors or parallelogram of forces covered in Modules IM2 and IM3.
- Perform the necessary laboratory procedure to find forces in the members of a given structure and verify by calculating the values to within ± 10% accuracy.

Bridges consist of structural members, as do roof trusses, box girders and steel framed buildings (Figure 1).

Each frame is to be considered as a series of structural members welded or pin-joined together in such a manner as to distribute the internal and external forces in the tension or compression loads (pull or push). Structures of this type resist static loads (dead loads) and dynamic loads (wind, cars, rain and snow). Tension members are called struts.

In Figure 2, a simple frame structure is shown. Note the ties and struts (arrows indicate direction of resistance by the members). If the member AB (or AC) were cut, A would fall toward B (or C), denoting that members AB and AC are in compression. Such members are called struts. However, if member BC is cut, B would move away from C, and we call this a tension member or tie.

If a series of similar structures are connected and pin-joined together, a familiar structure is constructed, i.e., a bridge or roof truss as shown in Figure 1a and 1b respectively.

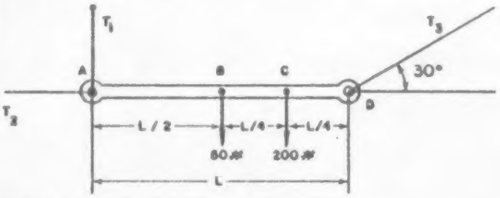
Similarly, other familiar structures are shown in Figures 3 and 4. These will be analyzed using vector and mathematical solutions.

Each of the previous figures illustrates what is considered to be a perfect structure, that is one that has just sufficient members in it to keep it in equilibrium under the known system of loading. This could also mean that it is the most economic structure in terms of cost.

All structural members must satisfy both conditions of mechanical equilibrium, i.e., $\Sigma F = 0$, $\Sigma \tau = 0$. By using either of the two methods covered in previous modules, graphical or mathematical (rectangular resolution method), the loads in a structural member can be found (see Examples A, B, C and D which follow). Consequently, size of the member can be chosen so that it can withstand the forces exerted upon it without deforming or breaking under the given load. Example A shows a simple structural member under the influence of three unknown forces T_1, T_2, and T_3. (Solution of this problem requires knowledge and formulas contained in Modules 1M2 and 1M3.)

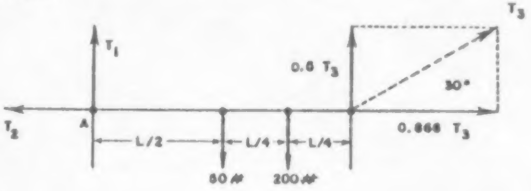
EXAMPLE A: SIMPLE STRUCTURAL MEMBER

Given: A uniform structural member (AD) with three rigid connecting members, T_1, T_2, and T_3 as shown in figure below. Each member weighs 50 lb. The member supports a load of 200 pounds at C.



Find: Tension in each member T_1, T_2, T_3.

Solution: To construct a force diagram, show the forces acting on the member AD and replace the force in T_3 by its vertical and horizontal components. This is shown below:



$\Sigma F_x = 0$:

$0.866\ T_3 - T_2 = 0.$ (1)

$\Sigma F_y = 0$:

$T_1 + 0.5\ T_3 - 50 - 200 = 0$ (2)

Choose point A as the rotational pivot as it will eliminate T_1 and T_2 from the equation of $(\Sigma\tau)_A$ and leave only T_3 to be solved.

From Solution figure of this example, $(\Sigma\tau)_A = 0$ gives

$$-50\ \frac{L}{2} - 200\ \frac{3L}{4} + 0.5\ T_3\ L = 0 \qquad (3)$$

Assume clockwise rotation is (-) and anticlockwise rotation is (+).

Force 0.86 T_3 is also eliminated because its line of action passes through the rotation at pivot point A and so has no lever arm.

Solve for T_3 by using Equation 3 after dividing both sides of the equation by L.

$$-25 - 150 + 0.5\ T_3 = 0$$

$$T_3 = 350 \text{ lb.}$$

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Substituting T_3 in Equation 1

$$0.866 \times 350 = T_2$$

$$T_2 = 303.1 \text{ lb.}$$

Substituting T_3 in Equation 2

$$T_1 = -0.5 \times 350 + 50 + 200$$

$$= -175 + 250$$

$$T_1 = 75.$$

As all calculated forces are positive, their directions were correctly chosen in figure under Solution of this example.

Check: Sum torques about another point B:

$$(\Sigma\tau)_B = 0$$

$$+ T_1L - 50(\tfrac{L}{2}) - 200(\tfrac{L}{4}) = 0$$

$$\therefore T_1 - \frac{50}{2} \quad \frac{-200}{4} = 0$$

$$75 - 25 - 50 = 0$$

$$75 - 75 = 0 \text{ (check).}$$

Example B shows the mathematical analysis of the loads in several members of a roof truss using a technique known as "method of sections." In this technique each point of the truss is isolated by cutting a plane through several of the members that meet at a selected point in such a fashion that no more than two unknown loads need to be found. From the basic equations of equilibrium, the horizontal and vertical forces may be resolved.

EXAMPLE B: ROOF TRUSS ANALYSIS

Given: A truss system as shown in figure at right.

Find: R_A, R_E, F_{AB}, F_{BH}.

Solution:

Total load on truss = 1500 + 1500

$$= 3000 \text{ lb (down)}.$$

This is supported by an upward reaction provided by the two supporting pillars. Because the bridge is symmetrical about the pillars,

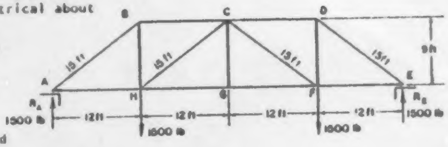
$$R_A + R_E = 3000 \text{ lb, and}$$

$$R_A = R_E$$

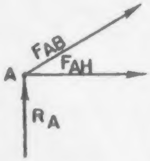
therefore

$$R_A = 1500 \text{ lb upward and}$$

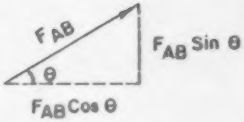
$$R_E = 1500 \text{ lb upward, as shown in the Figure.}$$

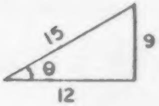


Now, joint A may be isolated by making the first cut through $\overline{AB}$ and $\overline{AH}$, and shown as a free body diagram as in figure at right.



F_{AB} can be resolved into horizontal and vertical components as shown. From the similar triangle showing dimensions of the truss, we have





$$\sin\theta = 9/15 = 3/5$$

$$\cos\theta = 12/15 = 4/5$$

The force F_{AB} in member AB can be resolved into:

Vertical component = $F_{AB} \sin\theta = 3/5\ F_{AB}$.

Horizontal component = $F_{AB} \cos\theta = 4/5\ F_{AB}$.

The vertical and horizontal conditions of equilibrium are then:

$$\Sigma F_H = 4/5\ F_{AB} + F_{AH} = 0.$$

$$\Sigma F_V = R_A + 3/5\ F_{AB} = 0$$

The second can be solved for F_{AB}:

$$3/5\ F_{AB} = -R_A = -1500$$

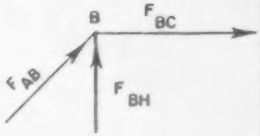
$$F_{AB} = 5/3\ (-1500) = -2500 \text{ lb}.$$

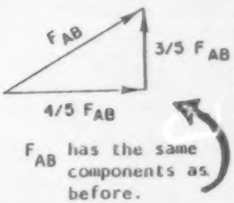
The negative sign means that the direction of F_{AB} is opposite that shown.

The other equilibrium equation now yields a value for F_{AH}:

$$F_{AH} = -4/5\ F_{AB} = -4/5\ (-2500) = 2000 \text{ lb}.$$

Next, joint B can be isolated and treated in a similar fashion. F_{AB} is now known to be 2500 lb. The force diagram is shown in margin.





F_{AB} has the same components as before.

Equilibrium conditions are:

$$\Sigma F_H = 4/5\ F_{AB} + F_{BC} = 0.$$

$$\Sigma F_V = 3/5\ F_{AB} + F_{BH} = 0.$$

Using F_{AB} = 2500 lb, these equations become:

$$2000 + F_{BC} = 0$$

$$1500 + F_{BH} = 0$$

which give: $F_{BC} = -2000$ lb.

$F_{BH} = -1500$ lt.

Again, the negative signs simply mean the directions assumed for these forces on junction B were incorrect and should be reversed. Their magnitudes, however, are correct. The desired answers are F_{AB} = 2500 lb and F_{BH} = 1500 lb. These strut members must be built to withstand the tensile forces calculated.

In this way, continue to consider points C, H, G, etc., until the tension in each truss member is calculated.

Example C shows the force diagram and a mathematical solution for a hinged fence gate that utilizes the second law of equilibrium, i.e., $\Sigma\tau = 0$.

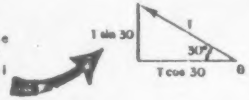
EXAMPLE C: HINGED FENCE GATE

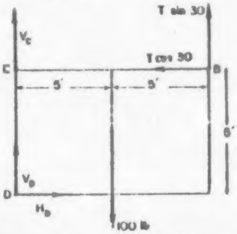
Given: A system shown in the diagram, assuming the uniform gate weighs 100 lb and there are no horizontal forces at hinge C.

Find: The system forces.

Solution: The force diagram will look like this:

Resolution of cable tension into horizontal and vertical forces.





Equilibrium of forces and torques acting on the gate require three conditions:

$$\Sigma F_H = H_D - T\cos 30^\circ = 0. \qquad (4)$$

$$\Sigma F_V = V_C + V_D + T\sin 30^\circ - 100 = 0. \qquad (5)$$

$$\Sigma\tau_D = (T\cos 30^\circ)5 + (T\sin 30)10 - (5) = 0. \qquad (6)$$

Equation 6 can be solved for T:

$$T\,(5\cos 30^\circ + 10\sin 30^\circ) = 500.$$

$$T\,(4.33 + 5) = 500.$$

$$9.33\,T = 500 \text{ or } T = 53.6 \text{ lb}.$$

Substituting this value into Equation 4 gives H_D:

$$H_D = T\cos 30^\circ = 53.6\,(0.866).$$

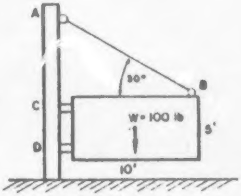
$$H_D = 46.4 \text{ lb}.$$

Finally, Equation 5 can be solved for (V_C + V_D), the sum of the vertical loads at the hinges:

$$V_C + V_D = 100 - T\sin 30^\circ = 100 - (53.6)(0.5).$$

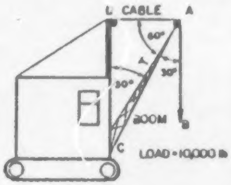
$$V_C + V_D = 73.2 \text{ lb}.$$

From this information for the loads, the hanging gate and post could be designed.



Example D covers the mathematical analysis of another familiar structure, a crane boom. In addition, this problem may be solved using the graphical methods covered in Module IM2, with particular reference to vector forces and triangle solutions.

EXAMPLE D: CRANE BOOM

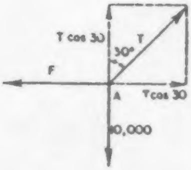


Given: Crane system shown in the diagram.

Find: Tension in the crane cables.

Solution: The analysis of a crane boom would be made as follows: Assume the crane is lifting a weight of 10,000 lb. Calculate the tension in the cable F_H when the crane boom is at 30° to the vertical, as well as the tension T in the boom.

A force diagram would look like figure in margin.



For equilibrium, $F_H = T \sin 30 = 1/2\ T$

$\quad$ and $T \cos 30 = 10{,}000$.

Then: $T = \frac{10{,}000}{0.866} = 11{,}547$ lb

and $F_H = 1/2\ T = 1/2\ (11{,}547)$

$\quad F_H = 5{,}774$ lb.

A crane boom is generally designed to withstand stresses of a maximum weight with a restricted angle of the boom to the vertical. If the boom were to exceed the angle, it could collapse due to higher bending stresses than it was designed for. As the crane boom angle increases, the load it is designed to carry must decrease. A safety alarm, fitted to the boom, will sound in the crane operator's cab when the limiting angle is approached.

When the load in a structural member has been ascertained, the member must be selected that can resist this load without deformation or failure. Module IM5, "Stress and Strain," will explain the selection of suitable members for this purpose.

Spring scale with a range of 0-2000 g.
Support stand.
Wooden dowel 60 cm long with hole drilled 10 cm from one end.
1.5 m length of fine strong wire or string.
Protractor.
Two 500 g weights.
Triple beam balance.

1. Weigh dowel and record mass in grams in Data Table.
2. Set up the apparatus as shown in Figure 5 using a weight of 500 g.
3. Adjust length of string containing spring scale to bring it to true horizontal.
4. Use the diagram of forces on the dowel shown in Figure 6 to develop the equations to solve for T and F.
5. Measure and record the actual value of T.
6. Repeat with a weight W_1 of 1000 g and a larger angle θ.

Case I

W_1 = ________ g ℓ_1 = ________ cm ℓ_3 = ________ cm
W_2 = ________ g ℓ_2 = ________ cm   θ = ________

Calculated values: F_x = ________ g
F_y = ________ g
F = ________ g
T = ________ g
Measured value: T = ________ g

Case II

W_1 = ________ g ℓ_1 = ________ cm ℓ_3 = ________ cm

W_2 = ________ g ℓ_2 = ________ cm θ = ________

Calculated values: F_x = ________ g

F_y = ________ g

F = ________ g

T = ________ g

Measured value: T = ________ g

1. Define:
   - Static load.
   - Perfect structure.
   - Strut.
   - Tie.
   - Frame structure.
2. Draw simple loading diagram for the following structural system, identify load and direction:

3. List the two conditions of equilibrium and apply them to the following structure.

4. Calculate the forces in the framed structure given in example 2 above, using methods of vectors.

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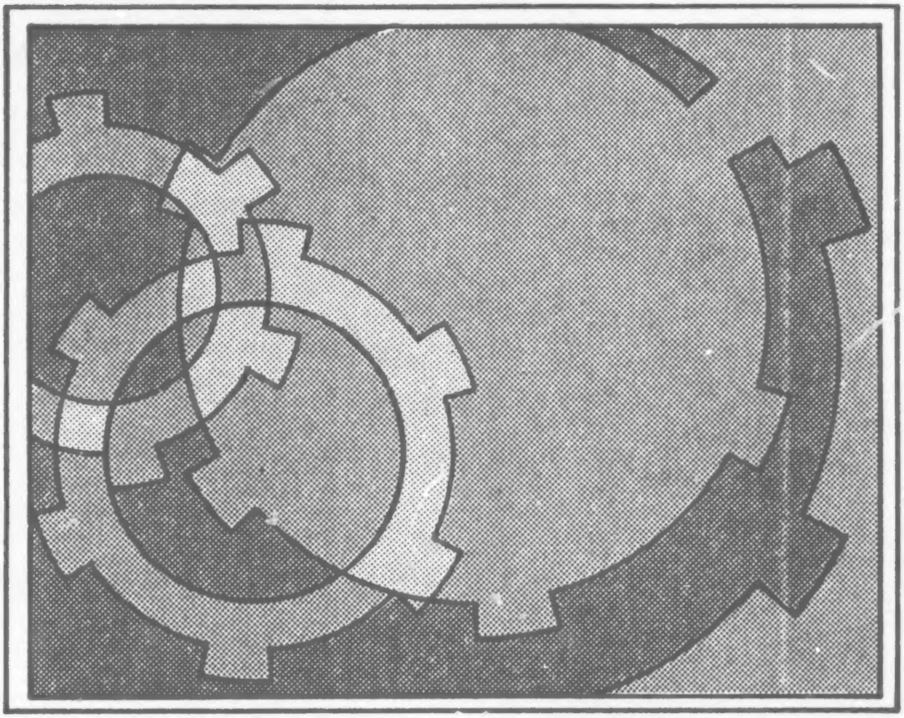
UNIFIED TECHNICAL CONCEPTS

MODULE 1 M 5

STRESS AND STRAIN



MECHANICAL APPLICATION OF FORCE

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Module IM4, "Structural Forces and Torques," develops the principles necessary for computing tensile and compressive forces in members of a structure. Practical use of this information requires knowledge of how large a member is needed to withstand these forces and how much the structure will distort under load. Elasticity is the study of the way actual materials are changed in shape by forces applied to them.

The elastic properties of materials are described in terms of two concepts known as "stress" and "strain." The concepts of stress and strain of elastic materials will be investigated in this module.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force," and Application Module IM2, "Addition of Mechanical Forces."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
    - Stress.
    - Strain.
    - Hooke's law.
    - Elastic limit.
    - Yield point.
    - Breaking point.
    - Modulus of elasticity.
    - Shear modulus.

  - Bulk modulus of elasticity.
  - Safety factor.
  - Ultimate breaking strength.
- Given appropriate forces and surface area of an object, calculate stresses produced.
- Given appropriate changes in dimensions of an object, calculate strain produced on that object.
- Given stress and strain on an object, derive the correct modulus of elasticity.
- Given the necessary equipment, perform the following laboratory procedures:
  - Experimentally verify Hooke's law.
  - Determine ultimate breaking strength of several different materials.
  - Measure deflection of a wood beam.

STRESS

If a body does not move away when a force or pressure is applied to it, it undergoes stress. No material is perfectly rigid, so the force will cause the material to deform. It will bend, twist, compress or change in some manner.

Stress depends on two factors: the force applied and the area over which the applied force is distributed. This relation is given by Equation 1.

$$\text{Stress} = S = \frac{F}{A} = \frac{\text{force applied}}{\text{area of application}}. \qquad (1)$$

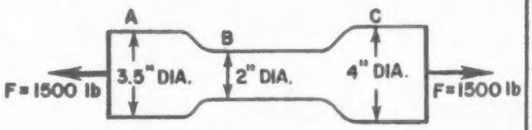
Example A illustrates use of this equation.

EXAMPLE A: STRESS PRODUCED IN A ROD OF VARYING DIMENSION

Given: A bar of size shown in diagram subjected to the applied loads.

Find: Stress produced in each section of the bar under tension.

Solution: The tensile force is the same for all three diameters = 1500 lb.



$$S_A = \frac{F}{A} = \frac{1500}{\frac{\pi}{4} \times (3.5)^2} = 156 \text{ lb/in}^2.$$

$$S_B = \frac{F}{A} = \frac{1500}{\frac{\pi}{4} \times (2)^2} = 477 \text{ lb/in}^2.$$

$$S_C = \frac{F}{A} = \frac{1500}{\frac{\pi}{4} \times (4)^2} = 119 \text{ lb/in}^2.$$

The area of each diameter at points A, B and C is different.

STRAIN

Strain measures change in dimension or dimensions due to some kind of stress. It is a pure number with no dimensions. Strain is defined by Equation 2.

$$\text{Strain} = e = \frac{\Delta \ell}{\ell_o} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{(\text{final length} - \text{original length})}{\text{original length}}$$

or

$$e_V = \frac{\Delta V}{V_o} = \frac{\text{change in volume}}{\text{original volume}}$$

$$= \frac{(\text{final volume} - \text{original volume})}{\text{original volume}}$$

$$e_\ell = \frac{\Delta \ell}{\ell_o}$$

or

$$e_V = \frac{\Delta V}{V_o}. \qquad (2)$$

Note: Strain can be (+) or (-) depending on the situation. Examples B and C illustrate the use of this equation.

EXAMPLE B: STRAIN IN A PIECE OF WIRE

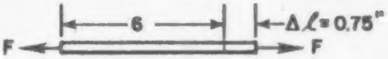
Given: A 6-inch wire increases to 6.75 inches when subjected to a certain force.

Find: Strain in the wind.

Solution: $e_\ell = \frac{\Delta \ell}{\ell_o}$

$= \frac{(6.75 - 6.00)}{6}$

$= 0.125.$



EXAMPLE C: STRAIN IN A CUBE OF METAL

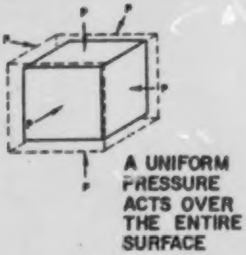
Given: Cube 10 inches on a side decreases to a cube 9.85 inches on a side when subjected to an external pressure.

Find: Strain in the cube.

Solution:

$$e_V = \frac{\Delta V}{V_o} = \frac{(9.85)^3 - (10)^3}{10^3} = \frac{956 - 1000}{1000}$$

$$= \frac{-44}{1000} = -0.044.$$



Therefore:

$$e_V = -0.044.$$

The negative sign indicates that the strain has produced a decrease in volume.

HOOKE'S LAW

Hooke's law states that, within limits, the stress divided by the strain is constant. This constant is called the modulus E and is defined by Equation 3.

$$\text{Modulus} = E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\Delta\ell}. \qquad (3)$$

One set of dimensions might be lb/in^2, but other units of force divided by area may be used.

The modulus for various materials is tabulted under "E" in Table 1.

ELASTICITY AND THE ELASTIC LIMIT

The modulus E is constant only over a certain range of loading for materials in tension. Beyond the point of critical loading, the material is no longer elastic. Increased tension will cause more rapid elongation and eventual breaking of the material (Figure 1).

TABLE 1. TYPICAL MECHANICAL PROPERTIES OF METALS						
	Ultimate Strength ($\times 10^3$ lb/in^2)					
Metal	Tension S_t	Compression, S_c	Shear S_s	Tension Yield S_y	Modulus of Elasticity, 10^6 lb/in^2 E	Modulus of Rigidity, 10^6 lb/in^2 G
Wrought iron	47	47	38	28	28	10.0
AISI 1020 steel (structural)	65	65	50	45	30	11.5
AISI 1045 steel	95	95	70	60	30	11.5
AISI 1095 steel	142	142	105	83	30	11.5
AISI 2340 steel	135	135	100	116	30	11.5
302 stainless steel	140	140	110	100	28	11.0
Cast iron class 20	20	80	32	---	11	4.5
Cast iron class 40	40	125	55	---	16	5.5
Cast iron class 60	60	170	65	---	19	8.0
Red brass	57	---	37	40	15	5.7
Yellow brass	61	---	40	50	15	5.8
17 S aluminum alloy	62	---	38	40	10.4	3.8
24 S aluminum alloy	68	---	41	46	10.4	3.8
Monel	80	---	56	40	26	9.5
AZ 31X magnesium alloy	40	---	19	30	6.5	2.4

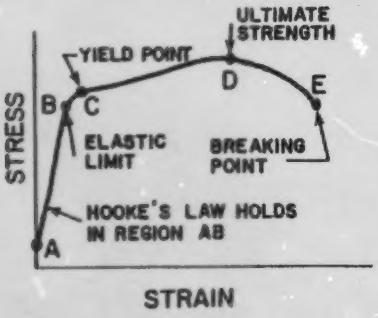


Fig. 1

The part of the curve between A and B is the proportional section of the curve. If at any point between A and B the stress is released, the material will return to its original length. Hooke's law is valid over this region.

When more heavily loaded (beyond B), the material will not return to its original length. The point B is called the "elastic limit."

Further increase in loading beyond B reaches point C which is the yield point. From there on, a small increase in loading stress gives a rapid elongation and strain.

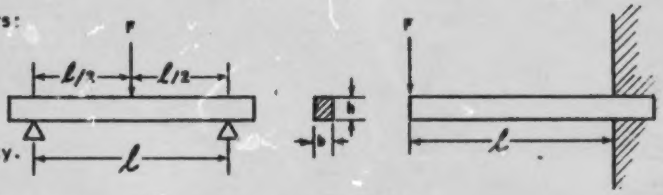
Next reached is point D, ultimate strength and finally, as load is further increased, the material breaks at point E. In the region from D to E, the cross-sectional area of the material is decreasing rapidly. This change must be considered in any calculations of stress for a material in region DE. However, this region is to be avoided in the design of structures.

DEFLECTION - MOVEMENT UNDER STRESS

Often the limiting factor in a design is not how much it is stressed or whether it will break and fail, but rather the deflection due to loading. This situation is exemplified by the frame of glass windows which must be rigid to prevent excess flexing and consequent breaking of the window panes.

In general, deflection depends on the size of the applied force, the modulus of elasticity E, and the length and cross section of the beam. Consider the two cases shown in Figure 2. Deflection, y, of a beam depends upon a number of factors:

F = force applied in pounds.
ℓ = length in inches.
E = modulus of elasticity.
I = "moment of cross section" = $bh^3/12$.



ENDS SUPPORTED BEAM CROSS SECTION OVERHANG-CANTILEVER

Fig. 2

Formulas giving the deflection are:

$y = \frac{F\ell^3}{48EI}$: Both ends supported, force at center, deflection at center. (4)

$y = \frac{F\ell^3}{3EI}$: Overhanging cantilever, force at end, deflection at unsupported end. (5)

These equations will be used later in a laboratory exercise.

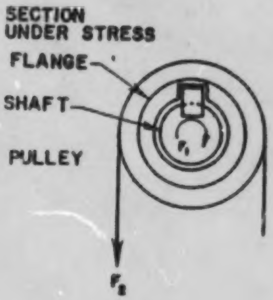


Fig. 3

SHEAR MODULUS

In the rotating shaft and pulley arrangement (Figure 3), there are different stresses and strains on the shaft, pulley and key. One of the major cross sections of maximum stress and failure is in the key and is shown by the dotted line A-B. This "shear" stress is caused by the force of the shaft, F_1 being resisted by the pulley force F_2. These forces tend to stress the material, causing the top half of the key to shear off the bottom half. Values of allowable shear stresses in various materials are shown in Table 1.

The formula for shearing stress is the same as that given by Equation 1, namely, the shearing force divided by the area under stress. Often the units are lb/in². Example D illustrates calculation of shearing stress.

EXAMPLE D: SHEAR STRESS IN AN ALUMINUM BLOCK

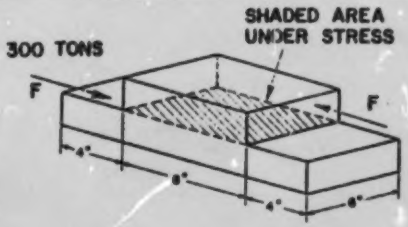
Given: A force of 300 tons applied as shown to the aluminum block. The top portion is fixed in place.

Find: Shearing stress. Will the allowable stress be exceeded? Can the block be used as shown? Assume 24S aluminum alloy (Table 1).

Solution: Actual stress $= \frac{F}{A}$

$= \frac{300 \times 2000}{8 \text{ in} \times 6 \text{ in}} = 12{,}500 \text{ lb/in}^2$.

(8 in x 6 in is the area under stress.)



Allowable stress from Table 1 is 41,000 lb/in². The block is usable in this application, because the actual stress, 12,500 lb/in² is well below the allowable.

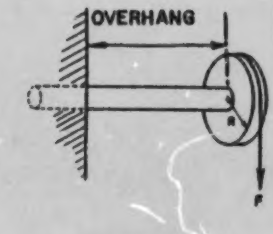


Fig. 4

SHAFTS IN ROTARY TORSION (TWIST)

The shaft shown in Figure 4 is fixed on the left and has a pulley with a force applied at radius, R. The shaft will have a torque stress induced in it, which can be calculated. The shaft will twist on the right end in the direction shown by the arrow. The angle of twist can be predicted by using a mathematical formula. If the turning force is too much and actual stress exceeds the allowable value, the shaft could break. If the shaft overhang is long, there will be a bending stress tending to break it in addition to the torque stress.

BULK MODULUS

Normal changes in air pressure do not perceptibly affect volume of a solid object. However, if a solid is acted on by a large pressure (for example, the pressure 1 mile deep in the ocean), it can be slightly compressed. The amount of compression is predicted by using the "bulk modulus of elasticity," B.

$$B = \frac{\text{stress}}{\text{strain}} = \frac{\Delta P}{\Delta V/V} = \frac{\text{extra pressure applied}}{\text{change in volume/original volume}}.$$

Typical values of B are listed in Table 2 and a calculation is shown in Example E.

TABLE 2. BULK MODULUS FOR SOLIDS AND LIQUIDS

Material	Bulk Modulus B	
	newtons/m^2 × 10^{10}	lb/in^2 × 10^6
Solids:		
Aluminum	7.7	11
Brass	6.1	8.5
Copper	14	20
Glass	3.7	5.3
Iron, cast	9.6	14
Lead	0.77	1.1
Steel	16	23
Liquids:		
Ethyl alcohol	0.110	0.16
Kerosene	0.13	0.19
Lubricating oil	0.17	0.25
Mercury	2.8	4.0
Water	0.21	0.31

EXAMPLE E: COMPRESSION DUE TO WATER PRESSURE

Given: An object two miles under the surface of the ocean, where added pressure is 4,600 lb/in^2.

Find: At this depth how much a cubic foot of brass would decrease in volume.
Note: 1 cubic foot equals 1,728 cubic inches.

Solution: Since,

$$B \text{ (from Table 2)} = \frac{\Delta P}{\Delta V/V}$$

$$\Delta V = \frac{\Delta P V}{B} = \frac{4600 \times 1728}{8.5 \times 10^6} = 0.94 \text{ in}^3.$$

The decrease of about 1 cubic inch would make the volume 1727 in^3. The % decrease in volume is 0.94/1728 or only 0.05%.

The compressibility, C, which is the reciprocal of the bulk modulus, B, is often referred to in the literature.

$$\text{Compressibility} = C = \frac{1}{B} = \frac{1}{\text{bulk modulus of elasticity}}.$$

LABORATORY

HOOKE'S LAW

MATERIALS

Hooke's law apparatus (Figure 5).
Set of metric weights.
Lengths of wire:
    Brass #14.
    Copper #14.
    Aluminum #10.
    Aluminum #18.
Micrometer or wire gage.
Graph paper.

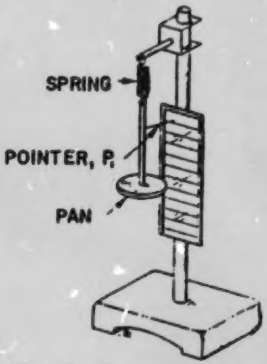


Fig. 5

PROCEDURE

1. Refer to Figure 5.
2. Set spring and pointer. Record scale reading opposite pointer with no load except pan; record in Data Table 1 as P_1 under trial 1.
3. Add a weight so pointer moves about 2 cm. Record weight and pointer reading (P_2).
4. Calculate stretch (S) which is $P_2 - P_1$.
5. Repeat steps 2 and 3 for increasing loads.
6. Plot a graph of stretch on horizontal axis versus weight on vertical axis as shown in Figure 6.
7. What is nature of curve? Does it agree with section AB of Figure 1?

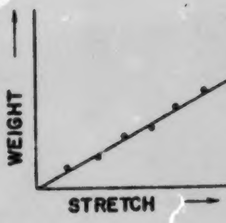


Fig. 6

DATA TABLE 1

Trial	Weight On Pan	Pointer Reading With No-Load P_1	Pointer Reading With Load P_2	Stretch of Spring $S = P_2 - P_1$
1	0 g			
2	g			
3	g			
4	g			
5	g			
6	g			

ULTIMATE BREAKING STRENGTH

MATERIALS

Set of metric weights.
Lengths of wire:
- Brass #14.
- Copper #14.
- Aluminum #10.
- Aluminum #18.

Micrometer or wire gage.
Ultimate strength tester (Figure 7).

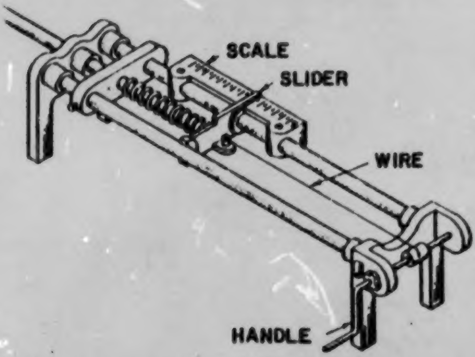


Fig. 7

PROCEDURE

1. Refer to Figure 7.
2. Measure diameter of a section of brass wire with micrometer; record in Data Table 2.
3. Place length of brass wire in machine. Turn handle until wire breaks.
4. In Data Table 2, record force indicated by slider at breaking point (Figure 8).
5. Repeat for copper and aluminum wires.
6. Calculate breaking strength by using Data Table 2.
7. Look up table value and compare.

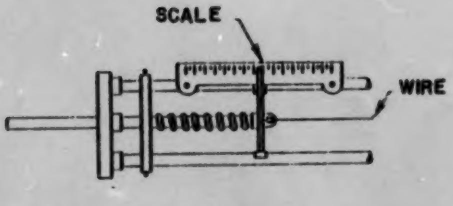


Fig. 8

DATA TABLE 2

Wire Material	Gage	Diameter (in)	Area = 0.785 D^2	Breaking Force, F (lb)	Strength = F/A	*Table Value

*Metric to English conversions may be required.

WOOD BEAM DEFLECTION

MATERIALS

Set of metric weights.
Flexure apparatus (Figure 9) including two knife-edge clamps, weight pan, micrometer screw with a 1/2-mm pitch and clamp to hold micrometer ridigly, test rods, bell and lead wires, weights: 1" x 2" pine beam approximately 1 yard long.

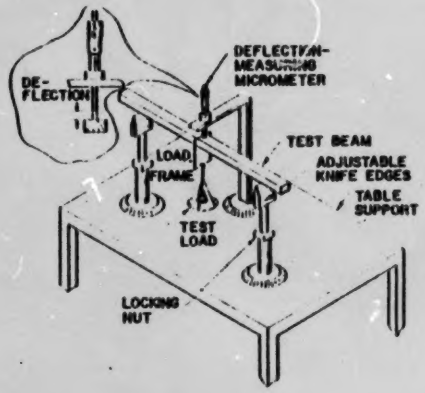

DE-
FLECTION
DEFLECTION-
MEASURING
MICROMETER
TEST BEAM
ADJUSTABLE
KNIFE EDGES
LOAD
FRAME
TABLE
SUPPORT
TEST
LOAD
LOCKING
NUT

Fig. 9

PROCEDURE

1. Refer to Figure 9.
2. Set up an end-supported wooden pine beam with $\ell = 20$ inches. Position beam so that its thickness in vertical direction is two inches. Load beam with weights; measure deflection. Double and triple weight; again record deflection (measurements 1A, 1B, 1C). See Data Table 3.
3. Repeat 1 with $\ell = 30$ inches (measurements 2A, 2B, 2C).

4. Repeat 3 with 1 inch beam dimension as vertical dimension (3A, 3B, 3C).
5. Set up a cantilever 25 in long. Load beam with weights; measure deflection. Double and triple weight; record deflection (4A, 4B, 4C).

DATA TABLE 3					
Trial	Applied F Pounds	Measured y Inches	ℓ Inches	$I = \frac{bh^3}{12}$	Calculated y From Formula (Equations 4,5)
1A			20		
1B					
1C					
2A			30		
2B					
2C					
3A			30		
3B					
3C					
4A			25		
4B					
4C					

SELF CHECK

1. Define:
    - Stress.
    - Strain.
    - Hooke's law.
    - Elastic limit.
    - Yield point.
    - Breaking point.
    - Modulus of elasticity.
    - Shear modulus.
    - Bulk modulus of elasticity.
    - Safety factor.
    - Ultimate breaking strength.
2. Find the stress placed on an object with an area of 8 in^2 subjected to 3000 pounds of force.
3. A plastic rod 6 inches long subjected to 2000 pounds of force changed the length to 5.73 inches, determine the strain that has been put on the rod.
4. Determine the modulus of elasticity of an object with 1500 lb/in^2 stress and 0.125 strain factor.
5. From the laboratory experiment on Hooke's law, answer the following questions.
    - What is the shape of the line that best fits the points plotted on the graph?
    - The graph shows that the stretch of the spring (strain) is __________ (inversely, directly) proportional to the applied force (stress).
    - If, after removing a weight from the pan, the spring scale does not return to the no-load reading, the __________ limit of the spring has been exceeded.
6. Answer the following questions based on observations of the beam deflection experiment
    - Does the deflection double and triple with double and triple weights?
    - By what approximate factor does deflection change when the beam is used with the 1-inch beam thickness h compared with the 2-inch thickness?
    - How do measured and calculated value of deflection compare?

REFERENCES

Sears, F.W. and Zemansky, M.W. College Physics. Reading, MA: Addison-Wesley Press, Inc., 1962.
See especially Chapter 11, "Mechanics, Heat, and Sound," page 192.

Williams, J.E.; Metcalfe, H. Clark; Trinklein, Frederick E.; and Lefler, Ralph W. Modern Physics. New York: Holt, Rinehart and Winston, 1968.
See especially Chapter 9, "Kinetic Theory of Matter," pages 183 - 187.

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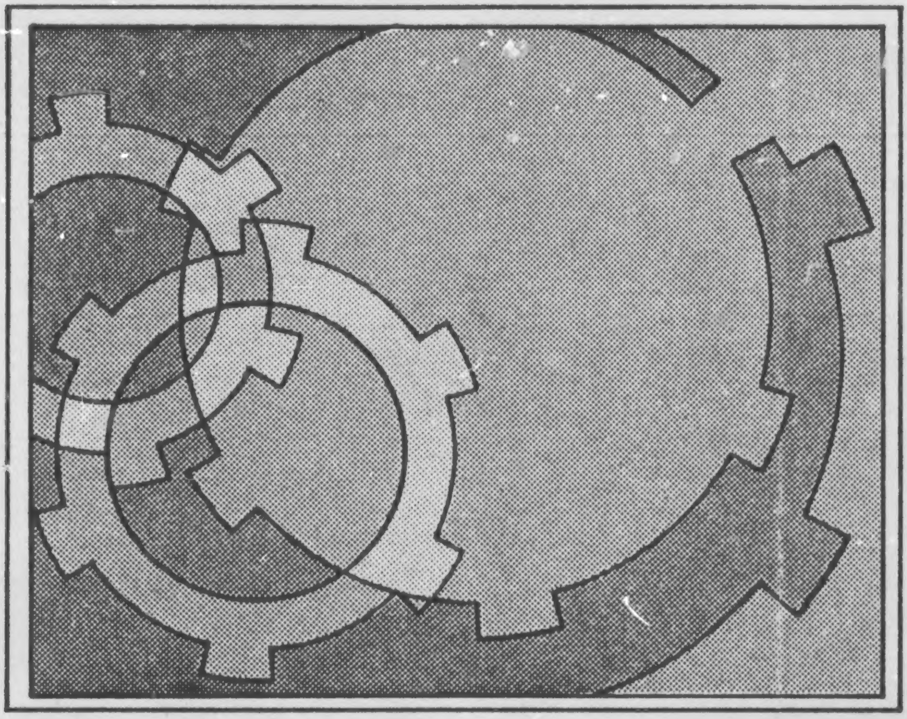
UNIFIED TECHNICAL CONCEPTS

MODULE 1 M 6

TENSION IN WIRES AND CABLES



MECHANICAL APPLICATION OF FORCE

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Forces are applied to objects to cause them to move or to resist impending motion. Cables, wires, strings and ropes are often used to apply force to an object. The force applied to objects using these flexible force agents is called tension. Automobiles, airplanes, botas and bicycles use cables to control the motion of the vehicles. Elevators, cranes, power lines and radio tower support lines have tension forces within them. This module discusses tension and its general characteristics and applications. Tension in weighted flexible cables, such as power lines, also is discussed. In the laboratory, the student will determine tension in a weighted line.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force," and Application Module IM2, "Addition of Mechanical Forces."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define a tension force.
- Describe the effects of a tension force applied to a wire, cable or rope.
- Given a set of cables attached to a load in such a manner that a concurrent force system is established, solve for the tension forces in all cables, using vector conditions of static equilibrium.
- Calculate the weight required to produce a given sag in a weighted cable.
- Given the appropriate equipment, determine tension at the center of a line and at the supports.

Forces are applied to many objects through cables, wires, ropes, strings and threads. (See Figure 1.) Man applies cables and strings, and uses forces to control, contain, or move various objects. They allow the transfer of forces over long distances. In industry, cables and ropes are so important in moving heavy equipment that an identifiable occupation has been named. Many large engineering companies employ "riggers." These men are in charge of controlling and preparing heavy loads to be moved by crane (Figure 2). To attach cables to several million dollars worth of heavy equipment that must be moved safely requires the expert knowledge possessed by riggers. They must know how to attach cables to loads and what safe working loads are.

FIG. 1

TENSION FORCES IN CABLES

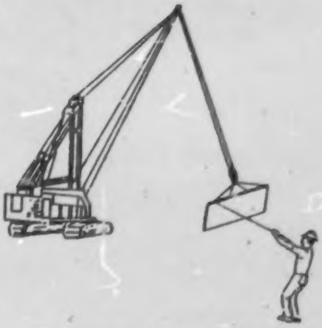


FIG. 2

A tension force tends to pull and stretch a body on which it acts. Actually, tension forces come in pairs and act together simultaneously upon the rope or cable. Figure 3 shows a tension force (pair) acting upon a rope segment. The tension forces are equal and oppositely directed. They are acting on the rope as well as within the rope. A frequent misinterpretation of tension forces arises when considering two forces pulling on a rope in opposite directions. For example, if there is a 100-lb force in opposite directions, a human tendency is to expect the tension in the rope to be 200 lb, although experiment would verify that the tension is 100 lb. Another important characteristic of tension in cables is that the direction of the tension force vector is always parallel to the direction of the central axis of the rope. This direction is the line of action, illustrated in Figure 3.

The line of action may be bent or the direction of tension forces in ropes and cables may be changed by use of a pulley. The pulley does not affect the magnitude of the tension force but affects only the direction of tension force (Figure 4).

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TENSION FORCES IN STATIC LOADING

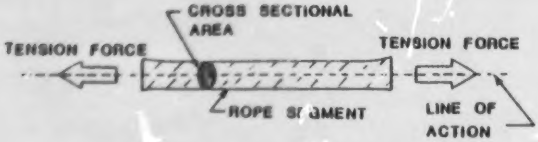


FIG. 3

Often, ropes and cables are attached to loads in such a manner that they form a concurrent coplanar force system. The forces involved in a concurrent coplanar force system are in the same geometrical plane and, when the rope or cable is stationary, the lines of action of all the forces intersect at a common point. The forces can be assumed to be acting at this point. An example of a concurrent force system is shown in Figure 5. The concurrency point is a key feature in identifying concurrent force systems. Note that in Figure 5 three forces are acting on a ring, and the point of concurrency is in the space at the center of the ring. Because the forces act through the same point, the load is prevented from rotating.

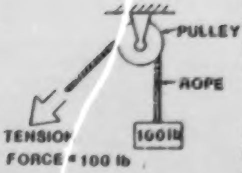


FIG. 4

When the load, as shown in Figure 5, remains stationary, it is in static equilibrium. When a system of forces is in equilibrium, the algebraic sum of the components of the forces along any mutually perpendicular coordinate axes is zero. For most static loading the vertical-horizonal coordinate axis system shown in Figure 5 is used. In this system, unknown elements (such as tension forces) of the system may be determined entirely by Equations 1 and 2 for force equilibrium.

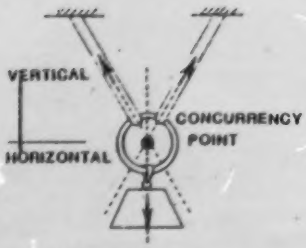


FIG. 5

The sum of all vertical components = 0

$$\Sigma F_V = 0 \tag{1}$$

(Σ is a mathematical symbol meaning "sum of.")

and

The sum of all horizontal components = 0

$$\Sigma F_H = 0 \tag{2}$$

Equations 1 and 2 are general equations that must be applied to the specific conditions of each individual problem to give equations which apply to that problem. To apply Equations 1 and 2, the horizontal and vertical components of each vector must be found. A sign convention is applied to each component. Upward-directed components are designated positive (+) forces; downward-directed components are designated negative (-) forces. Similarly, horizontal force components directed to the right are considered

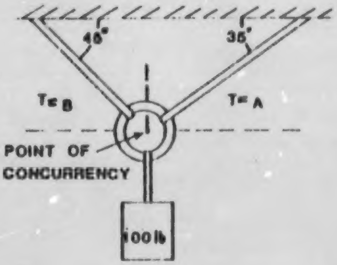
(+) forces, and those to the left (-) forces.

Equations 1 and 2 are used to solve a problem in Example A.

EXAMPLE A: EQUILIBRIUM OF CONCURRENT FORCE SYSTEM

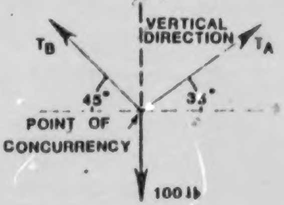
Given: A 100 lb weight supported by two ropes, A and B, at the angles shown.

Find: The tension in Rope A and Rope B. Note: Neglect weight of the rope.


45°
35°
T= B
T= A
POINT OF CONCURRENCY
100 lb

Solution:

1) As an aid, draw a vector diagram illustrating all forces acting on the point of concurrency. Label the magnitudes of the forces on the diagrams, giving letter names for unknown values. Label all known angles.


T_B
VERTICAL DIRECTION
T_A
45°
35°
POINT OF CONCURRENCY
100 lb

VECTOR DIAGRAM

2) Resolve all vectors into horizontal and vertical components. Write component equations for each direction, even though these equations may contain unknown values:

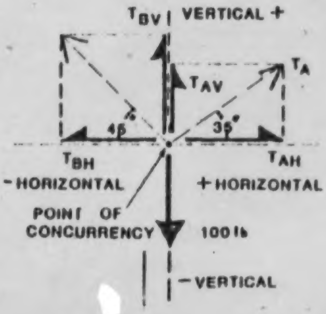
HORIZONTALLY:

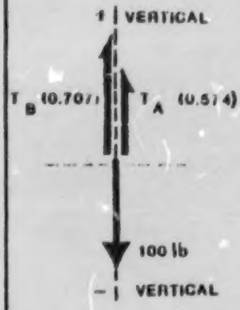
$$T_{AH} = T_A \cos 35° = T_A\ (0.819)$$
$$T_{BH} = T_B \cos 45° = T_B\ (0.707)$$

VERTICALLY:

$$T_{BV} = T_B \sin 45° = T_B\ (0.707)$$
$$T_{AV} = T_A \sin 35° = T_A\ (0.574)$$

Known force = 100 lb


T_BV
VERTICAL +
T_A
T_AV
45°
35°
T_BH
T_AH
- HORIZONTAL
+ HORIZONTAL
POINT OF CONCURRENCY
100 lb
- VERTICAL



3) By using Equation 1,

The sum of all vertical components = 0,

focus attention on vertical direction only. Write an equation representing the sum of all vertical forces, observing force sign convention:

$$\Sigma F_V = T_{BV} + T_{AV} - WT = 0$$

$$T_B\,(0.707) + T_A\,(0.574) - 100\text{ lb} = 0$$

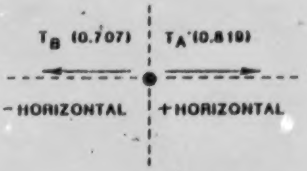
This equation is a specifc one, true for this problem. Note that it contains two unknowns, T_B and T_A, the desired tension forces. The horizontal direction furnishes an additional needed equation.

4) In the horizontal direction, the forces are as follows:

$$\Sigma F_V = -T_{BH} + T_{AH} = 0$$

$$-T_B\,(0.707) + T_A\,(0.819) = 0$$

According to this equation, horizontal components balance each other.



5) Solve the horizontal equation for T_B.

$$T_B = \frac{(0.819)\,T_A}{(0.707)} = (1.16)\,T_A$$

6) Substitute T_B above into the specific vertical equation.

$$+\,[(1.16)\,T_A]\,(0.707) + T_A\,(0.574) - 100\text{ lb} = 0$$

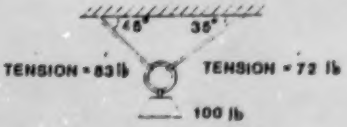
$$T_A = 71.7\text{ lb}$$

7) Substitute this value in the equation of Step 5,

$$T_B = \frac{(0.819)}{(0.707)}\ (71.7\ \text{lb})$$

$$T_B = 83\ \text{lb}$$

8) In summary, the tension in rope A is 72 lb, and the tension in rope B is 83 lb.

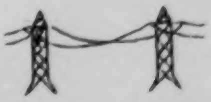


The steps for finding tension forces in cables of a concurrent force system can be summarized as follows:

- Identify the point of concurrency in the problem.
- Draw a sketch of the problem, labeling unknown and known forces.
- Draw a vector diagram.
- Resolve all forces into horizontal and vertical components.
- Apply the general conditions of equilibrium to develop specific equations for the problem.
- Solve for the values of the unknowns.

FLEXIBLE CABLES , HORIZONTALLY-UNIFORM LOADS

Telephone wires, power wires, and trolley wires whose ends are lying at or near the same level are examples of uniform, horizontal distribution of load weight. In such distribution, shown in Figure 6, load on the cable is described as weight per unit length. Sag produced in the cable is due to the cable's own weight distributed along the length between the supports. The amount of sag (distance the cable drops down from the support level at its lowest point) is affected also by the initial tension placed on the wire. For metal power lines, sag also is affected by temperature changes, which cause the lines to sag more in hot weather than in cold weather. In hot weather, lines expand and increase in length, therefore, sagging more. In cold weather, lines contract and decrease in length, therefore, sagging less.



POWER LINES

FIG. 6

Tension force in a cable is always parallel to the cable axis, but unlike earlier tension forces, it varies in magnitude along the cable length. Figure 7 illustrates the variables involved in describing the tension forces in flexible cables. The cable is suspended between two supports a distance ℓ apart. The tension

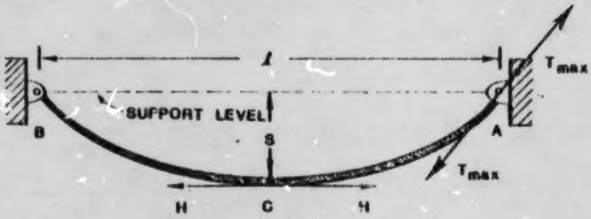


FIG. 7

varies in magnitude from H at the center C to a maximum of T_{max} at the ends of cable supports A and B. The sag S is measured from the support level to point C. The horizontal tension at the bottom of the cable sag is given by Equation 3.

$$H = \frac{W\,\ell^2}{8\,S} \qquad (3)$$

where

H = horizontal tension.
W = weight distribution per unit length (lb/ft, n/m, etc.).
ℓ = distance between supports.
S = sag.

The maximum tension (T_{max}) in the cable at the supports is given by Equation 4.

$$T_{max} = \frac{W\,\ell}{2}\sqrt{\frac{\ell}{16\,S^2} + 1} \qquad (4)$$

Example B illustrates the use of Equations 3 and 4 in determing the tension in a flexible cable.

EXAMPLE B: TENSION IN A POWER LINE

Given: A power line weighing 2 lb per ft is stretched between two supports at equal elevations 100 ft apart. The sag at the center is 10 ft.

Find: Tension H at the center and maximum tension at one support.

Solution: 1) Using Equation 3:

$$H = \frac{W\,\ell^2}{8\,S}$$

$$H = \frac{(2\text{ lb/ft})(100\text{ ft})^2}{8\,(10\text{ ft})}$$

$$H = 250\text{ lb} = \text{horizontal tension at center}$$

2) Using Equation 4:

$$T_{max} = \frac{W\,\ell}{2}\sqrt{\frac{\ell^2}{16\,S^2} + 1}$$

$$T_{max} = \frac{(2\ \text{lb/ft})(100\ \text{ft})}{2}\sqrt{\frac{(100\ \text{ft})^2}{16\,(10\ \text{ft})^2} + 1}$$

T_{max} = 269 lb = maximum tension at supports

In actuality, all cables, wires, ropes and strings possess a uniform distributed load; however, when the weight of a cable or rope is very small compared to the lumped load or weight attached to them, the weight of the cable or rope is usually neglected in a force analysis.

SUMMARY

Tension is the force which exists in cables, wires and other lines under load. The calculation of tension, sag and weight applied to lines has practical applications in power lines, phone lines, in general all lines bearing a load.

TENSION IN A WEIGHTED LINE

MATERIALS

Two vertical support stands.
Two thermometer supports.
20 meters of heavy manila twine.
A sheave or pulley.
Weight hanger.
Set of slotted weights.
Meter stick.
Metric steel tape (30 m).
Triple beam balance.

PROCEDURE

1. Arrange experimental apparatus as shown in Figure 8.

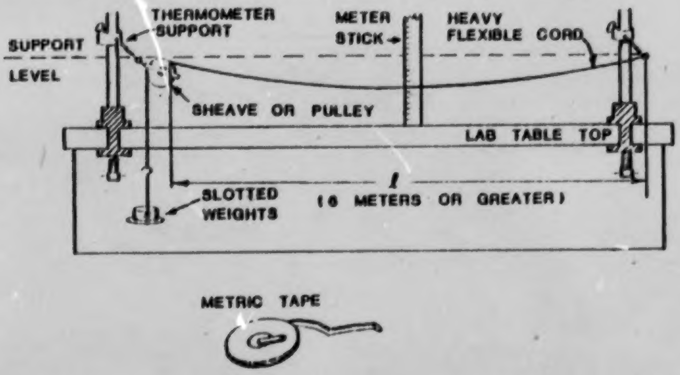

THERMOMETER SUPPORT
METER STICK
HEAVY FLEXIBLE CORD
SUPPORT LEVEL
SHEAVE OR PULLEY
LAB TABLE TOP
ℓ
SLOTTED WEIGHTS
(8 METERS OR GREATER)
METRIC TAPE

FIG. 8

2. Place a 100 g mass on 50 g weight hanger. Pull weight hanger down a small distance and release. This procedure allows the system to come to equilibrium.

3. Measure a 10 m segment of heavy twine or cord. Roll into a small, secure roll, and place it on the beam balance. Record mass (kg) of 10 m segment in Data Table.

4. Measure distance from thermometer stand on one end to top of pulley at other end. Record this distance, ℓ, in the Data Table.

5. Measure amount of sag using meter stick. This measurement should occur at half the distance between supports. Record amount of sag in Data Table.

6. Divide the mass in kilograms obtained in step 3 by the 10-meter length to obtain the mass per unit length. Multiply by acceleration of gravity (g = 9.8) to obtain weight per unit length in newtons/meter.

7. Using Equation 4, calculate maximum tension in cord at top of the pulley. Record this value in Data Table.

8. Record total weight of loaded mass plus mass of weight hanger. (Determine weight in Newtons by multiplying number of kilograms by g, the acceleration of gravity.)

9. Calculate % difference between results of step 7 and results of step 8.

10. Repeat steps 4 through 9 for several larger separation distances of supports and loaded masses.

11. How is tension in the line related to sag, distance between supports, and weight?

DATA TABLE

Distance Between Supports, ℓ (m)	Sag in Cable at Center, C (m)	T_{max} Calculated (N)	Load on Hanger Plus Hanger (N)	% Difference

Mass of 10 m length = __________ kilograms
Weight per unit length, W = mg = __________ N/m

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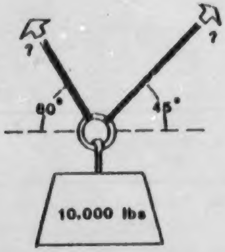
SELF CHECK

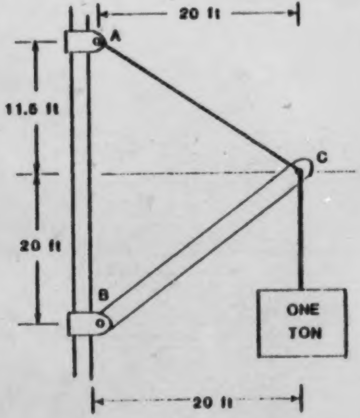
1. Define a tension force.

2. List two effects on a wire, cable or rope when a tension force is applied.

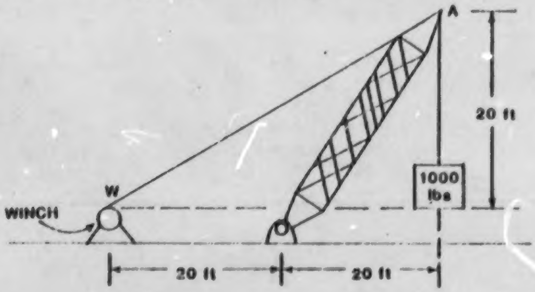
3. Two cables are used to lift a load of 10,000 lb. One cable pulls to the right, making a 45° angle with the ground. The other cable pulls to the left, making a 60° angle with the ground. Find the tension forces in the cables.



4. A simple derrick is shown which supports a load of one ton. Calculate the tension force in cable AC. Neglect weight of cable and strut BC.

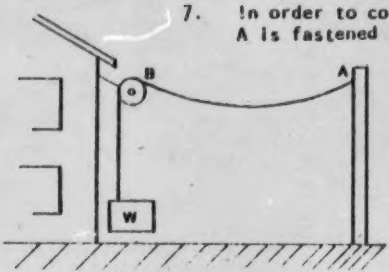


5. The simple arrangement shown must support a 1000 lb vertical load by means of a rope attached at A. What is the tension in rope AW? Neglect weight of the rope.



6. Compute the maximum tension in a flexible wire weighing 0.4 lb per ft stretched between two supports 200 ft apart and at equal levels, if the sag is 10 ft.

7. In order to control the sag in a radio aerial wire, one end A is fastened to the top of a tower, as shown in the accompanying figure, while the other end passes over a small sheave B and supports a weight W. If the wire weighs 0.05 lb/ft, and if A and B are 80 ft apart and at the same level, compute the weight W required to keep the sag at 2 ft. Neglect the diameter of the sheave B and the weight of the wire between B and W.



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Jensen, Alfred and Chemoweth, Harry H. Statics and Strength of Materials. 2nd ed. New York: McGraw-Hill Book Company, 1967.

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UNIFIED TECHNICAL CONCEPTS

MODULE 1 F 1

ATMOSPHERIC PRESSURE

FLUID APPLICATION OF FORCE

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

The earth is surrounded by a blanket of air that we call the "atmosphere." The atmosphere, a mixture of gases, has weight. Although the weight of a given volume of air is small when compared with the weight of an equal volume of almost any solid or liquid, still the total atmosphere exerts a considerable force per unit area on all objects at the surface of the earth. This force is about 15 pounds on every square foot; thus, atmospheric pressure is a physical quantity that plays an important role.

In this module, measurement of atmospheric pressure with barometers is presented. The pressure of a confined gas also is studied. In the laboratory, the student will construct a mercury barometer and will determine the pressure and volume of confined air.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Draw a simplified mercury barometer, and in two or three sentences explain how it functions.
- Briefly describe the use of an open-tube manometer to measure gage pressure and the closed-tube manometer to measure absolute pressure of gases.
- Give a one or two sentence description of gage pressure, absolute pressure, and atmospheric pressure; and show their mathematical relationship.
- Recite Boyle's law, and express it in mathematical equivalents.

- Given a confined gas at constant temperature, use Boyle's law to calculate pressure with a known volume or volume with a known pressure.
- Given the appropriate equipment, construct a mercury barometer, and determine the pressure and volume of confined air.

AIR PRESSURE

Atmospheric pressure is a result of the weight of air pushing on a surface. Pressure of any fluid, including air, varies with depth and weight density of the fluid according to Equation 1.

$$p = \rho_W h \tag{1}$$

where

p = pressure in lb/in^2.
ρ_W = weight density in lb/ft^3.
h = height in feet.

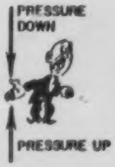


FIG. 1

Another interesting fact about fluid pressure is that, at a given depth in the fluid, pressure is applied equally in all directions. The man in Figure 1 feels no net pressure on his hand, because the pressure down on the top of it is equal to the pressure up on the bottom.

If a vacuum pump were positioned to remove the air under his hand, however, pressure difference would be noticed as is shown in Figure 2.

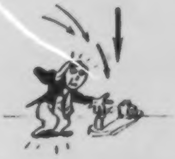


FIG. 2

Normal atmospheric pressure (pressure at sea level) is approximately 14.7 lb/in^2. The man in Figure 2 notices a total force of approximately 440 lb on his hand. This force was determined by the assumption that an "average" hand has a surface area of approximately 30 square inches.

PRESSURE MEASUREMENT

Two instruments commonly used to measure atmospheric pressure are the mercury barometer and the aneroid barometer. The principal part of an aneroid barometer is a low-pressure chamber with a flexible surface (Figure 3). Movement of this surface causes a dial to move, indicating the correct pressure on a calibrated scale.

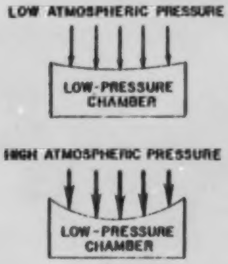


FIG. 3

The main principle upon which the mercury barometer operates is shown in Figure 4. The weight of a column of air many miles high is balanced by the weight of a column of mercury less than a meter tall.

No one could attempt to perform the measurement indicated in Figure 4. Figure 5 shows realistic forms of the mercury barometer.

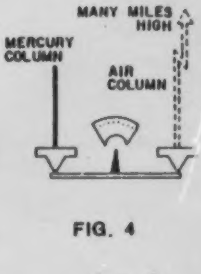


FIG. 4

In either a or b of Figure 5, the weight of a column of mercury of height h exactly balances the weight of the atmosphere; that is, pressure due to the weight of a column of mercury of height h is equal to atmospheric pressure p_o. Liquid pressure determined by the column of height h is given by Equation 2.

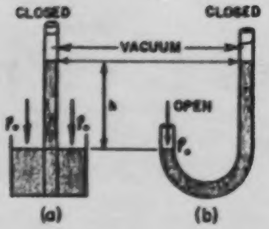


FIG. 5

$$p_o = \rho g h \qquad (2)$$

where

p_o = liquid pressure.
ρ = mass density of liquid.
g = acceleration due to gravity (9.8 m/s^2 or 32 ft/s^2).
h = height of liquid column.

As the pressure exerted by the liquid column is exactly equal to the atmospheric pressure at the base of the column, $p_o = p_{atmospheric}$. Notice that, as $\rho g = p_W$, the weight density, this equation is equivalent to Equation 1.

Example A shows the use of Equation 2 and how to convert "centimeters of mercury" to other pressure units.

EXAMPLE A: STANDARD ATMOSPHERIC PRESSURE

Given: Under standard conditions (sea level at 0°C), the height of a column of mercury in a barometer is 76 cm.

Find: Pressure in dyne/cm².

Solution:

$$p_o = \rho g h$$
$$= (13.6\ g/cm^3)(980\ cm/s^2)(76\ cm)$$
$$p_o = 1.013 \times 10^6\ dyne/cm^2$$

Note: 1 dyne is the force necessary to increase the velocity of a 1 gram mass by 1 cm/s. 10^5 dynes = 1 Newton. Thus, if the atmosphere "balances" a column of mercury 76 cm high, pressure exerted by the atmosphere is equal to about one million dynes/cm².

At higher altitudes, air becomes rarer; and the "depth" of atmosphere is less, so units in Table 1 are at sea level. For example, atmospheric pressure at the top of Mt. Everest is only about 27 cm of mercury.

TABLE 1: VARIOUS UNITS FOR STANDARD PRESSURE

Units	Pressure Equals
SI	1.013×10^5 newton/meter^2
Cg/s	1.013×10^6 dyne/cm^2
British	2120 lb/ft^2
Practical	14.7 lb/in^2
	76.0 cm of mercury
	29.9 inches of mercury

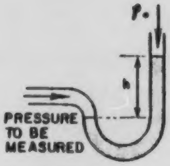


FIG. 6

Gas pressures can be measured conveniently with an open-tube manometer. The manometer (Figure 6) is a U-shaped tube containing a liquid, such as mercury or water. The upper end is open to the atmosphere, and the other end is connected to the gas for which the pressure is to be measured. Evidently, the gas pressure is balanced by the pressure due to the height h of liquid and the atmospheric pressure p_o. The height h is referred to as the "gage pressure." Total, or absolute, pressure of the gas then is given by Equation 3.

$$p = p_{gage} + p_a \qquad (3)$$

where

p = absolute, or total, pressure.
p_{gage} = gage pressure.
p_a = atmospheric pressure.

Examples B and C illustrate use of open-tube manometers in determining gage and absolute pressure of a confined gas.

EXAMPLE B: OPEN-TUBE MANOMETER AND STEAM PRESSURE

Given: One end of a manometer is connected to a steam boiler. Difference in the levels of mercury is 28 cm. Assume atmospheric pressure to be 76 cm of mercury.

Find: Pressure of the steam (absolute pressure).

Solution:

$$p = p_{gage} + p_a$$
$$= 28 \text{ cm} + 76 \text{ cm}$$
$$p = 104 \text{ cm}$$

EXAMPLE C: OPEN-TUBE MANOMETER AND LUNG PRESSURE

Given: Lung pressure of a patient is determined by blowing into one end of a manometer containing water. (Mercury is poisonous.) The gage pressure is 60 cm of water. Assume p_o to be 74.6 cm of mercury.

Find: Absolute lung pressure.

Solution: As density of mercury is 13.6 g/cm³, and density of water is 1 g/cm³, the height of 1 cm of mercury should weigh the same as the height of 13.6 cm of water.

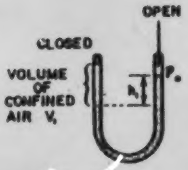
$$h = (60 \text{ cm of water})\left(\frac{1 \text{ cm of mercury}}{13.6 \text{ cm of water}}\right)$$

$$h = 4.4 \text{ cm of mercury}$$

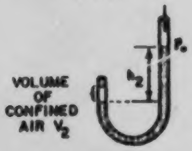
Absolute lung pressure:
(Remember that $p_o = p_a$.)

$$\begin{aligned} p &= p_{gage} + p_a \\ &= 4.4 + 74.6 \end{aligned}$$

$$p = 79.0 \text{ cm of mercury}$$



POSITION 1



POSITION 2

FIG. 7

Figure 7 displays a manometer that has one end open and the other closed. Air is trapped in the closed end. The closed-tube manometer can be used to determine pressure of the confined air. Assume that the tube is flexible and that it can be shifted from position 1 to position 2. In position 1, initial gage pressure is h_1, so initial absolute pressure on the confined air, p_1, is equal to $h_1 + p_o$. Volume of the confined air is V_1. In position 2, gage pressure has been increased to h_2, and the new absolute pressure is equal to $p_2 = h_2 + p_o$. Thus, p_2 is greater than p_1. As Figure 7 indicates, the volume of confined air, V_2, will be reduced from volume V_1 when a greater pressure p_2 acts on it. This relationship is consistent with Boyle's law, which states the following:

> If the temperature of a confined gas remains constant, the pressure acting on the confined gas is inversely proportional to the volume of the gas.

The following formula is an inverse proportion, and is a mathematical statement of Boyle's law:

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

By cross multiplication, the formula can be written in the more convenient form shown in Equation 4.

$$p_1V_1 = p_2V_2 \qquad (4)$$

where

p_1 = initial absolute pressure on confined air.

p_2 = later and greater absolute pressure on confined air.

V_1 = volume of confined air at p_1.

V_2 = volume of confined air at p_2.

Boyle's law also can be stated thusly:

$$pV = k \qquad (5)$$

where

k = proportionality constant.

Example D illustrates (1) use of the closed-tube manometer to measure volume of a confined gas and (2) use of Equations 4 and 5 in solving a problem.

EXAMPLE D: CLOSED-TUBE MANOMETER

Given: A mercury barometer indicates atmospheric pressure as 75 cm of Hg. A manometer is set up as in Figure 7. Initially, the mercury in the open side is 10 cm higher than that of the closed side. Initial volume of the enclosed air is measured to be 30 cm^3.

Find: Volume of the enclosed air if the left side is (a) Lowered until the mercury in it is 27 cm below the mercury in the right side; (b) raised until the mercury in it is 15 cm above the mercury in the right side; and (c) raised until the levels of mercury are equal.

Solution: (a) Gage pressure: h_1 = 10 cm (position 1)
h_2 = 27 cm (position 2)

$p_1 = h_1 + p_0$ $\qquad$ $p_2 = h_2 + p_0$

p_1 = 10 + 75 = 85 cm of mercury $\qquad$ p_2 = 27 + 75 = 102 cm of mercury

From Boyle's law: $p_2V_2 = p_1V_1$

$$\begin{aligned} p_2V_2 &= p_1V_1 \\ 102\ V_2 &= (85)(30) \\ 102\ V_2 &= 2550 \\ V_2 &= 25\ \text{cm}^3 \end{aligned}$$

Notice that, regardless of position, the product of the pressure and the volume will always be the same:

$$p_4V_4 = p_3V_3 = p_2V_2 = p_1V_1 = \text{constant} = k$$

Use Boyle's law in the form of Equation 5:

$$pV = k$$

(In this case, k = 2500; parts b and c can be solved.)

(b) Gage pressure is -15 cm of mercury:

$$\begin{aligned} p &= (-15) + 75 = 60\ \text{cm} \\ pV &= \text{constant} \\ 60\ V &= 2550 \\ V &= 42.5\ \text{cm}^3 \end{aligned}$$

(c) Gage pressure is zero:

$$\begin{aligned} p &= 0 + 75 = 75 \\ 75\ V &= 2550 \\ V &= 34\ \text{cm}^3 \end{aligned}$$

SUMMARY

Atmospheric pressure is the force exerted by the atmosphere upon, and for many miles above, the earth's surface. Such pressure is greatest at the surface, where it is about 15 pounds per square inch. Atmospheric pressure is measured by barometers.

MAKING A MERCURY BAROMETER

MATERIALS

Boyle's law apparatus, such as Sargeant Welch #1080.
Two or three pounds of mercury.
Stopcock greaser.

PROCEDURE

PRECAUTIONS — PLEASE NOTE: Handling mercury should be avoided because it is highly toxic, irritating to the skin, and may permanently discolor jewelry. Before opening a stopcock, consider whether or not the sudden change in pressure will cause the mercury to spill out of the tube. Every effort should be made to clean up any mercury spilled on the floor.

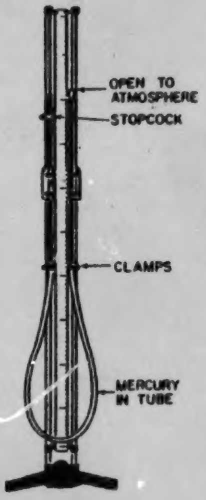


FIG. 8

1. The U-shaped tube shown in Figure 8 is known as a Boyle's law apparatus. It is partially filled with mercury. Tube is secured on both sides with clamps so that either side can be raised or lowered.
2. Before opening stopcock, adjust apparatus so that mercury levels in tube are approximately equal.
3. Open stopcock. With both sides of tube now open to atmosphere, levels will be exactly equal.
4. Lower stopcock side (or raise open side) until mercury comes right up to stopcock.
5. Close stopcock; raise stopcock side (or lower open side) further until there is a space between stopcock and mercury. This space will constitute a vacuum, because there is no way for air to get in (unless stopcock leaks). If stopcock leaks, level of mercury will be observed to continue dropping gradually. In this event, repeat procedures 2 and 3 after applying some stopcock grease to stopcock.
6. When no air occupies space between top of mercury on left side and stopcock, measure how much higher one side of mercury column is above other. Record

this difference in Data Table as p_o (atmospheric pressure) in cm of mercury.

7. Explain how this barometer operates.

PRESSURE AND VOLUME OF CONFINED AIR - BOYLE'S LAW

MATERIALS

Boyle's law apparatus, such as Sargeant Welch #1080.
Two or three pounds of mercury.
Stopcock greaser.
Vernier calipers (optional).
Graph paper.

PROCEDURE

1. Using same U-shaped tube, raise one side and lower other side until mercury levels are about the same. Open stopcock.

2. Raise or lower appropriately until mercury level on stopcock side is about 20 or 25 centimeters below stopcock. Close stopcock; leave it closed for rest of experiment.

3. With apparatus in this position (refer to this as position 1); measure the gage pressure; record this in centimeters of mercury. Attach a positive or negative to gage pressure, depending upon whether height difference h is positive or negative (Example D and Figure 7 of Discussion).

4. Calculate absolute pressure (Equation 3) of trapped volume of air. Record this pressure.

5. With apparatus in position 1, measure length of trapped air in closed tube. This is distance from top of mercury to bottom of stopcock. Record this distance in centimeters.

6. To find true volume of trapped air, measure inside diameter of tube with vernier calipers; record. Calculate volume of trapped air by the following equation.

$$V = \pi r^2 h$$

where

π = Pi, 3.14.
r = inside radius (not diameter) of tube.
h = length of trapped air measured in procedure 5.

Record this volume in cm^3.

7. Multiply pressure and volume together; record result as proportionality constant k, as shown in Equation 5.

8. Vary levels of mercury to new positions (position 2); repeat procedures 3-7.

9. Repeat for three other positions (3, 4, 5).

10. From these five independent measurements, calculate an average proportionality constant k. Record and use this k in further calculations.

11. Choose three different values for absolute pressure, different from those used above. Record under "absolute pressure (student's choice." Calculate volume in each case using Equation 5. Record under "Volume (calculated)."

12. Choose three values for volume, different from those used above. Record under "Volume (student's choice)." Calculate absolute pressure and gage pressure in each case; record.

13. Student should now have eleven pairs of values for *p* and V. Make a graph of pressure versus volume. Draw a smooth curve through these points (Figure 9). This curve will not be a straight line. It should look more like a sagging rope. It is actually part of a hyperbola with *p* and V-axes as asymptotes.

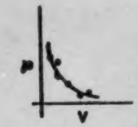


FIG. 9

14. Choose three values of pressure different from any one of eleven values above. Record these under "Absolute Pressure." From curve, estimate values of volume for each of these pressures; record under "Volume (from curve)."

DATA TABLE

Position Of Apparatus	Length of Enclosed Air Column (cm)	Gage Pressure h (cmHg)	Absolute Pressure p (cmHg)	Volume Of Enclosed Air Column (cm^3)	Proportionality Constant k
1					
2					
3					
4					
5					
			(Student's Choice)	(Calculated)	Average Value
		(Calculated)	(Calculated)	(Student's Choice)	
			(Student's Choice)	(Estimated From Curve)	

Atmospheric pressure p_o = _______ cmHg

Inside diameter of tube = _______ cm

Volume of enclosed air column = _______ cm^3

SELF CHECK

1. Draw a mercury barometer, and explain how it functions.

2. Describe the use of an open-tube manometer to measure gage pressure and of the closed-tube manometer to measure absolute pressure of gases.

3. Define and give the mathematical relaitionship of the following terms.

   (a) gage pressure.
   (b) absolute pressure.
   (c) atmospheric pressure.

4. State Boyle's law, and express it in mathematical equivalents.

5. What volume of oxygen gas at atmospheric pressure will fill a 2.5-ft^3 tank to a pressure of 2000 lb/in^2?

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See especially Chapter 13.

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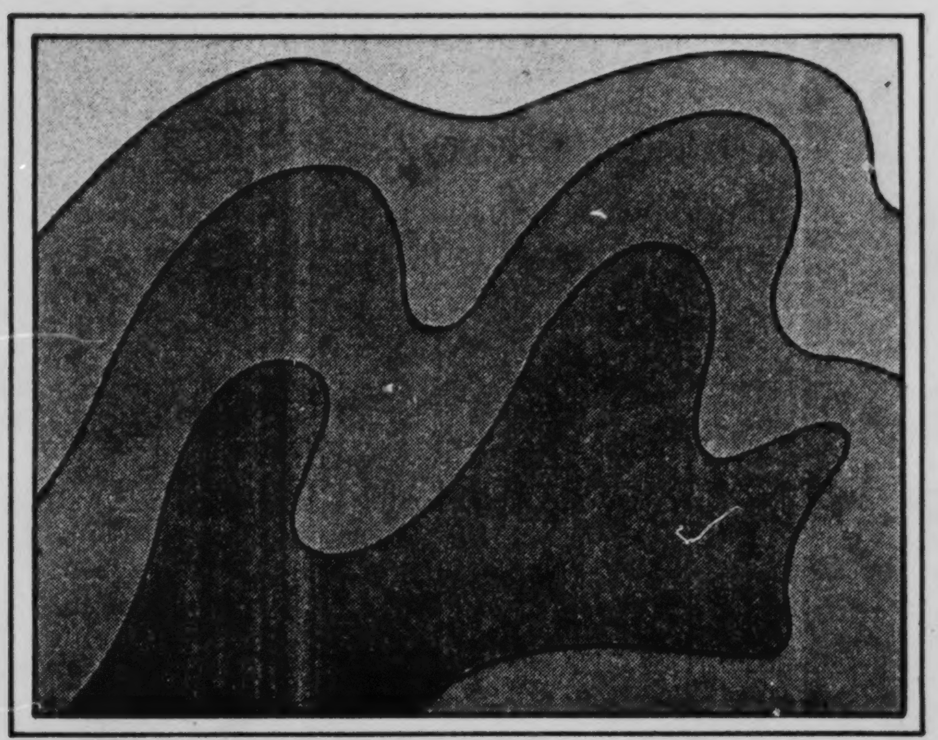
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 1 F 2

DENSITY, BUOYANCY, AND SPECIFIC GRAVITY



FLUID APPLICATION OF FORCE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

This module introduces the methods of determining the density and specific gravity of fluids by means of floating objects placed within those fluids. Such suspended objects are acted upon by the downward force of gravity (weight) and by the upward buoyant force of the fluid. Specific gravity is applied in determining the amount of antifreeze in radiators, the amount of acid in batteries, etc. In the laboratory, the student will use a hydrometer to determine the specific gravity of several fluids.

PREREQUISITES

Student should have completed Concept Module I-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- State Archimedes' principle.
- State and apply formulas for density and specific gravity.
- Explain what causes the byoyant force on a floating object.
- Explain how one can determine specific gravity and density of an object that sinks in water.
- State, explain and apply formula for specific gravity and density of liquids and floating objects.
- Describe the function and operation of a hydrometer.
- Given the appropriate equipment, determine the specific gravity of several fluids.

DISCUSSION

DENSITY

Figure 1 shows a wooden cube 2 cm on each side with a mass of 4 grams. Its volume is 8 cm^3 (found by multiplication of length × width × height: 2 cm × 2 cm × 2 cm = 8 cm^3). Density is determined by one of the formulas shown in Equation 1. One possible set of units for each is given.

$$\rho_m = \frac{m}{V}$$

or

$$\rho_W = \frac{W}{V} \qquad (1)$$

where

ρ_m = mass density.
m = mass in grams.
V = volume in cm^3.
ρ_W = weight density.
W = weight.

This wooden block with a density of 0.5 g/cm^3 is one-half as dense as water. If the block is placed into water, it will float with one-half the block above water and one-half the block under water. It will be buoyed up by the water.

BUOYANCY

Weight of fluid displaced equals buoyant force on any object floating or submerged in a fluid. If the block is floating in water (Figure 1), the bottom will be at a depth of 1 cm. The buoyant force at that depth is found simply by multiplying volume of the water displaced (4 cm^3) by the weight density of water. (1 cm^3 of water has a mass of 1 gram, and 1 gram weighs 980 dynes.) Note: 10^5 dynes = 1 Newton.

Buoyant force = $(4\ cm^3)(1\ g/cm^3)(980\ \frac{dynes}{g})$
Buoyant force = 3920 dynes.

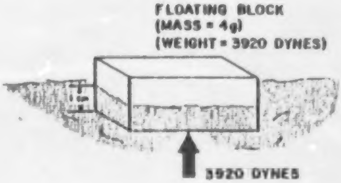


FIG. 1

Objects that do not float are also buoyed up by a force equal to the weight of the water displaced. A 10-pound fish can be pulled to the surface with an 8-pound test line; but a dip net must be used to lift the fish from the water, because the weight of the fish above water (10 lb) would snap the 8-pound test line.

An object in a fluid experiences two forces: One force is the downward force due to gravity, which is simply the weight of the object; the other force is the upward buoyant force of the fluid. For the object to float, there must be an upward force exactly equal to the downward force. Figure 2 illustrates a common instance in which the buoyant force causes an object to float.

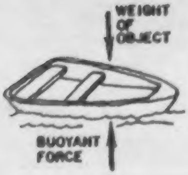


FIG. 2

These examples illustrate Archimedes' principle, which states the following:

> A body partially or entirely submerged in a fluid will experience a buoyant force equal to the weight of the fluid displaced.

The king had given Archimedes the task of determining whether or not his crown was made of pure gold. While taking a bath, Archimedes realized that impure gold would lose a greater proportion of its entire weight than would pure gold when submerged in water. It is said that he was so excited with this discovery that he jumped up from the tube and ran through the streets yelling, "Eureka! I've got it, I've got it!"

USING BUOYANCY IN DETERMINING DENSITY

Now that the same block referred to earlier is placed in methanol antifreeze, a liquid having a density of 0.80 g/cm^3. To support the 4-gram block, the buoyant force must equal its weight, 3920 dynes, as shown before. But the block will sink lower in the methanol antifreeze than in the water, because a larger volume of the less-dense methanol must be displaced to equal 3920 dynes. The formula for density and Archimedes' principle is applied to find the volume of displaced fluid as follows:

$$\text{Weight of displaced fluid} = \text{weight of floating block}$$

$$\text{Vol of displaced methanol} \times \text{weight density} = \text{weight of floating block}$$

$$V \times (0.80 \times 980)\ \frac{\text{dynes}}{\text{cm}^3} = (4 \times 980)\ \text{dynes}$$

$$V \times 784\ \frac{\text{dynes}}{\text{cm}^3} = 3920\ \text{dynes}$$

$$V = \frac{3920\ \text{dynes}}{784\ \text{dynes/cm}^3} = 5.0\ \text{cm}^3$$

As 3920 dynes of methanol antifreeze must be displaced by the 4-g block for the block to float, volume of the fluid displaced must be 5 cm^3. This means that 5 cm^3 of the 8-cm^3 block (5/8 or 62.5%) lies below the surface.

In practice, one may need to determine the mass or weight density of a fluid knowing the fraction of the object lying above or below the fluid surface. Suppose the block is floating in a fluid of unknown density. If it is determined that 5/8 or 62.5% of the block is below the surface, weight density of the fluid can be calculated from Equation 1 as follows:

$$\rho_W = \frac{W}{V}$$

where

W = weight of fluid displaced.
V = volume of fluid displaced.

Since the mass of the block is 4 g, and since the block is in equilibrium (therefore supported by a buoyant force of 4 x 980 dynes), weight of the fluid displaced (according to Archimedes' principle) is 3920 dynes. Then the weight density is as follows:

$$\rho_W = \frac{W}{V}$$

$$= \frac{3920 \text{ dynes}}{8 \text{ cm}^3 \times 0.625} \quad \text{(As 62.5\% is below the surface.)}$$

$$\rho_W = 784 \frac{\text{dynes}}{\text{cm}^3}$$

Mass density is determined as follows:

$$\rho_m = \frac{m}{V}$$

$$= \frac{784 \text{ dynes/cm}^3}{980 \text{ dynes/g}}$$

$$\rho_m = 0.80 \frac{\text{g}}{\text{cm}^3}$$

Example A shows another kind of problem that uses the Equation 1 density formula in its solution.

The density of other mixtures could be determined by use of similar calculations.

EXAMPLE A: DENSITY OF A MIXTURE

Given: A car with a 16-quart capacity radiator. It is filled with a 50%/50% mixture of ethylene glycol (antifreeze) and water; that is, 8 quarts antifreeze and 8 quarts water. Mass density of water is 1 g/cm^3. Mass density of ethylene glycol is 1.11 g/cm^3. One quart = 946.5 cm^3.

Find: Mass density of the mixture.

__Solution__: Use Equation 1 for mass density:

$$\rho_m = \frac{m}{V}.$$

$$\rho_{mixture} = \frac{\text{mass of mixture}}{\text{volume of mixture}}$$

mass of mixture = mass of water + mass of ethylene glycol

m_w = mass of water = (density of water)(volume of water)

$$m_w = (1\ g/cm^3)(8\ qt)(946.5\ \frac{cm^3}{qt}) = 7572\ g$$

m_e = mass of ethylene glycol = 1.11 g/cm^3 (8 quarts)(946.5) = 8405 g

$$\rho_{mixture} = \frac{7572 + 8405\ g}{(16\ qt)(946.5\ cm^3/qt)}$$

$$\rho_{mixture} = 1.055\ g/cm^3$$

Thus, density of the 50-50 mixture is 1.055 g/cm^3.

SPECIFIC GRAVITY

Specific gravity of a substance is a number comparing density of the substance to density of water. This relationship is shown in Equation 2.

$$S.G. = \frac{\rho_s}{\rho_w} \qquad (2)$$

where

S.G. = specific gravity of a substance.
ρ_s = density of substance.
ρ_w = density of water, 1.0 g/cm^3.

In the metric system of units, as density of water is 1.0 g/cm^3, the specific gravity of a substance will be equal numerically to the density of the substance. Specific gravity has no units, because it represents a ratio of two numbers with like units. As specific gravity has no units, it must have the same value, regardless of the system of units in which one is working.

Sometimes a more convenient approach is to simplify Equation 2 and to compare the weights or masses of equal volumes of two substances rather than their densities. The ratio still will be the same, as here indicated:

FIG. 3

$$S.G. = \frac{\rho_s}{\rho_w} = \frac{W_s}{W_w}$$

$$S.G. = \frac{m_s}{m_w}$$

where

W_s = weight of substance.
W_w = weight of equal volume of water.
m_s = mass of substance.
m_w = mass of equal volume of water.

A hydrometer such as the one shown in Figure 3 often is used to measure specific gravity of liquids.

The hydrometer is a glass bulb weighted and calibrated in such a manner that the level at which it floats in a fluid will indicate directly the specific gravity of the liquid. Density of the liquid is determined from the scale in the units — g/cm^3.

Most antifreeze testers in commercial use have hydrometer scales divided into two parts. The upper scale is used for water-methanol mixtures (specific gravities from 0.79 to 1.00), and the lower scale is used for water and ethylene-glycol mixtures (specific gravities from 1.00 to 1.16).

Specific gravity of antifreeze is important, because it gives an indication of the concentration of the mixture of antifreeze and water. On the back of an antifreeze container, the freezing temperatures for different mixtures of antifreeze and water are listed. Table I indicates the freeze-protection temperatures for antifreeze protection.

TABLE I: APPROXIMATE TABLE FOR ANTIFREEZE PROTECTION

Type	Parts Of Water	Parts Anti-freeze	Ratio	Specific Gravity	Freezing °F	Temperature °C
Water	16	0	--	1.0	32°	0°
Methanol	6	10	3 to 5	0.0875	-67	-55
	8	8	1 to 1	0.90	-38	-39
	10	6	5 to 3	0.925	-15	-26
	12	4	3 to 1	0.95	+5	-15
Ethylene-glycol	6	10	3 to 5	1.062	-65	-54
	8	8	1 to 1	1.055*	-34	-37
	10	6	5 to 3	1.041	-8	-22
	12	4	3 to 1	1.027	+10	-12

*As calculated in Example A.

Battery testers are the same as antifreeze testers, with a little additional weight to the hydrometer bulb. They measure specific gravity of the battery acid and water mixture (in most cases sulfuric acid). The scale indicates specific gravities from 1.10 to 1.30. If the reading is between 1.26 and 1.30 (colored green), the battery is charged. If the reading indicates less than about 1.23 (red) the battery is discharged.

A laboratory method for determining specific gravity of a solid object often is used; it is based upon Equation 3.

$$\text{S.G.} = \frac{m_A}{m_w} \quad \text{or} \quad \text{S.G.} = \frac{W_A}{W_w} \tag{3}$$

where

m_A = mass of object.
m_w = mass of an equal volume of water.
W_A = weight of object.
W_w = weight of an equal volume of water.

The specific gravity of objects that sink in water is given by Equation 4.

$$\text{S.G.} = \frac{W_A}{W_A - W_w} \tag{4}$$

Note that W_A is the weight of the object and that $(W_A - W_w)$ is the weight of an equal volume of water. The latter claim follows because the loss of weight of the object in water $(W_A - W_w)$ is equal to the weight of water displaced, by Archimedes' principle. As the object was totally submerged, this is the weight of the body's volume of water. Hence Equation 4 follows at once from the general definition for specific gravity.

A similar method can be used to determine specific gravity of a given fluid by use of Equation 5.

$$\text{S. G.} = \frac{W_A - W_L}{W_A - W_w} \tag{5}$$

where

W_L = weight of an object submerged in an unknown liquid.

Because $W_A - W_L$ is the loss of weight when the object is submerged in the given liquid, it will also equal the buoyant force in that liquid. From Archimedes' principle, $W_A - W_L$ must be the weight of the liquid displaced by the object; therefore, $W_A - W_L$ divided by $W_A - W_w$ is the ratio of weight of the liquid displaced to weight of the same volume of water displaced. This ratio, of course, is equal to specific gravity of the given liquid.

SUMMARY

When an object is suspended in a fluid, the object is acted upon by two forces: (1) the force of gravity (weight) downward, and (2) the force of buoyance upward. The specific gravity of a fluid can be determined, therefore, by the degree of suspension of an object in the fluid. The hydrometer contains an object in a calibrated tube; the level at which the object floats indicates the specific gravity of the fluid tested.

DETERMINING SPECIFIC GRAVITIES OF FLUIDS WITH A HYDROMETER

MATERIALS

Laboratory balance.
Meter stick.
Block of wood such as a short piece of 2 x 4 (not a cube).
Three containers (preferably transparent) large enough for the wooden block to float with its large surface in a horizontal position.
Ethylene-glycol and water mixture (about 1/2 gallon).
Methanol (methyl alcohol) and water mixture (about 1/2 gallon).
Laboratory hydrometer (capable of reading specific gravity from 0.8 to 1.2).
Automobile radiator hydrometer (antifreeze tester).

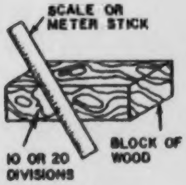


FIG. 4

PROCEDURE

1. Weigh block of wood on a laboratory balance. Record in Data Table.

2. Measure length, width, and height of block of wood; calculate its volume. Record.

3. With a scale slanted, as shown in Figure 4, side of block can be divided into 10 or 20 divisions. Mark these divisions with a pencil.

4. Arrange three containers of liquids as shown in Figure 5.

5. Place block of wood in container A. Count number of divisions below surface; divide by total number of divisions; multiply by 100. This calculation gives percent of block below surface. (Note: Block should not be left floating long enough to become water-logged.)

6. Calculate volume of block below surface. (Multiply total block volume by decimal equivalent of percent below surface.) Record.

7. Record buoyant force. (It must be equal to weight of block, as block is floating and is therefore in equilibrium.)

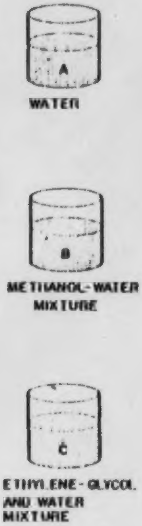


FIG. 5

8. Record weight of fluid displaced. (Apply Archimedes' principle.)

9. Record volume of fluid displaced (same as volume of block below surface).

10. Calculate fluid density.

11. Repeat for fluids in containers B and C.

12. Use hydrometer to determine specific gravity of each fluid. Convert this reading to density for each fluid.

13. Find percent error in each case, assuming density as determined with hydrometer to be correct.

14. Using value of density obtained with hydrometer, look up approximate freezing temperature for fluids A, B and C in Table 1.

15. Use antifreeze tester to measure freezing temperature of fluids A, B and C.

16. Why were the hydrometer balk suspended higher in some fluids than in others?

DATA TABLE

Weight of wooden block _______.

Length _______. Width _______. Height _______. Calculated Volume _______.

	A	B	C
Percent of block below surface			
Volume of block below surface			
Buoyant force			
Weight of fluid displaced			
Volume of fluid displaced			
Density of fluid			
Density measured by hydrometer			
Percent error			
Approximate freezing temperature (from table)			
Approximate freezing temperature (from antifreeze tester)			

SELF CHECK

1. In your own words, describe Archimedes' principle.

2. Is this a correct formula for density? If not, what must be changed?

$$\rho_m = \frac{V}{m}$$

3. Would it be possible to lift a 12-pound weight from the bottom of a lake to the surface with a line of 10-pound test strength? Why or why not?

4. If the weight of an object and the weight of an equal volume of water were known, could the specific gravity be determined? If not, what other information would be needed? If so, describe the solution in an equation format.

5. The specific gravity of aluminum is 2.7. The weight of water is 62.4 lb/ft^3. Calculate the weight in pounds of a two cubic foot solid piece of aluminum.

6. A popular antifreeze tester is the kind that has several little balls, each having a slightly different density. A sample is taken from a radiator in which ethylene-glycol is used, and all of the balls are found to be floating.

   Would the fluid in the radiator be in danger of freezing?

   Explain your answer. You probably will wish to use these terms: buoyant force, weight of displaced fluid, and density (or specific gravity).

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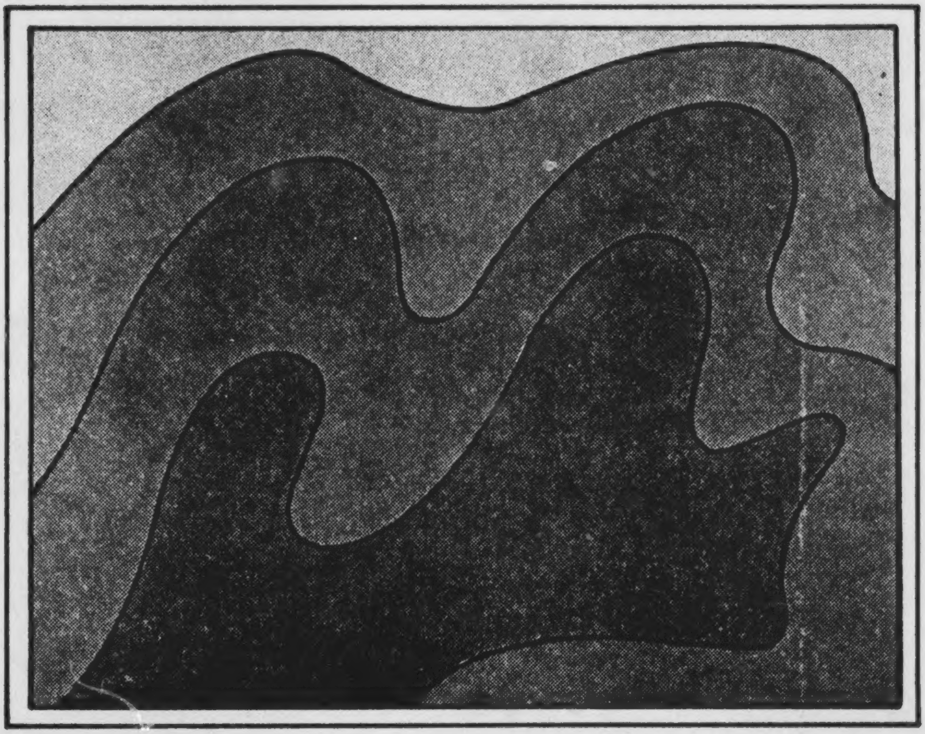
UTC
UNIFIED TECHNICAL CONCEPTS

MODULE 1 F 3
HYDRAULIC JACK



FLUID APPLICATION OF FORCE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

The hydraulic jack is a device commonly used to lift heavy objects with relative ease. It uses the principle of the force transformer, such as the lever that allows use of a relatively small force to lift a large weight. In this module, the student will be introduced to operating principles of a hydraulic jack and the role of force and fluid pressure in the operating characteristics of the actual hydraulic jack.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force," and should be able to measure linear distance accurately with vernier caliper.

OBJECTIVES

To complete this module, the student should be able to:

- In a short paragraph define a closed hydraulic system.
- Recite Pascal's law and illustrate it by giving the equation that describes it in an idealized hydraulic system.
- Define an idealized hydraulic system in one or two sentences.
- Describe in a short paragraph how a hydraulic jack amplifies force.
- Briefly define and illustrate with an equation the "breakaway force" in a hydraulic jack.
- Describe the operation and role of one-way check valves in a hydraulic jack.

DISCUSSION

The hydraulic jack is a simple device used to lift heavy loads. Typically it is used by automechanics to lift autos and trucks. The hydraulic jack pictured in Figure 1 is a representative example of a closed hydraulic system. Another example of a closed hydraulic system is the braking system on an automobile.

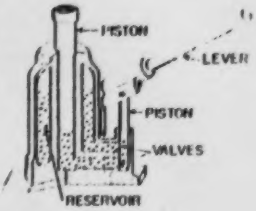


Fig. 1

In a closed hydraulic system, volume remains constant. It is usually filled with an incompressible liquid that will not significantly change volume when placed under pressure. Liquids such as water, oil, or brake fluid have this characteristic, and are frequently called "hydraulic liquids." Since incompressibility of the hydraulic fluid is a desirable characteristic, it is also important not to have air bubbles in the hydraulic fluid. Air (or any other gas) is easily compressed. In addition, for proper functioning of a closed hydraulic system, the reservoir should not possess air leaks.

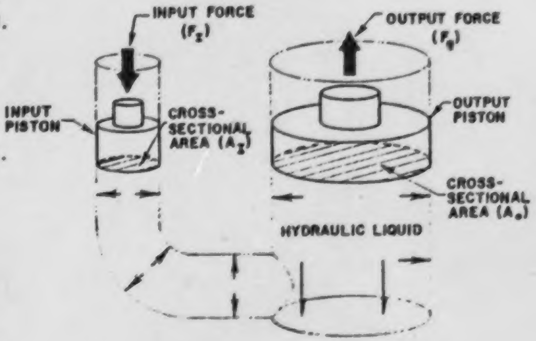


Fig. 2

To describe the physical principles upon which a hydraulic jack operates, the idealized two-piston system, shown in Figure 2, will be studied. The pistons shown are separated by an incompressible fluid. The load to be lifted by the hydraulic jack is supported on the large piston, while input force is supplied by the hand-operated lever system shown in Figure 1. An input force (F_I) is applied to the small piston of cross-sectional area (A_I). This force is transmitted by the piston to the liquid as a fluid pressure (F_I/A_I as defined in Concept Module 2-0). Pressure F_I/A_I, created by the input piston, is then transmitted throughout the hydraulic fluid and is the same throughout the entire volume of the fluid. This principle was first observed by the famous scientist Blaise Pascal. Pascal's law can be stated as:

> In a closed system filled with an incompressible fluid, pressure applied to one part of the fluid is transmitted equally to all parts of the system.

Relating Pascal's law to the idealized hydraulic system:

$$\text{Pressure at input piston surface} = \text{pressure at output piston surface.} \qquad (1)$$

The output piston, however, has a much larger surface area (A_o). Therefore a proportionately larger force will be experienced at the output piston.

If pressures in Equation 1 are expressed in terms of forces and areas, the following equality results:

$$\frac{\text{input force}}{\text{area of input piston}} = \frac{\text{output force}}{\text{area of output piston}}$$

or

$$\frac{F_I}{A_I} = \frac{F_o}{A_o}.$$

Rearranging this algebraically, the equation for output force F_o becomes:

$$F_o = \left(\frac{A_o}{A_I}\right) F_I. \qquad (2)$$

The objective of the experiment in this module will be to verify Equation 2. Example A illustrates the use of Equation 2.

EXAMPLE A: HYDRAULIC SYSTEM PROBLEM

Given: A hydraulic jack having an input piston with 1 in^2 surface area and an output piston with 15 in^2 surface area.

Find: Ratio of the output force to the input force.

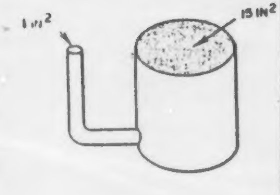
Solution:

$$\frac{\text{Output force}}{\text{input force}} = \frac{\text{area of output piston}}{\text{area of input piston}}$$

$$= \frac{(15\ in^2)}{(1\ in^2)}$$

$$\frac{\text{Output force}}{\text{input force}} = \frac{15}{1}.$$

For example: A 10-pound force exerted downward on the small piston will raise a 150-pound weight resting on the large piston.



The preceding discussion of an idealized hydraulic jack assumed that all applied input force was used to increase pressure in the liquid. In actual hydraulic systems, applied force must overcome certain frictional forces in addition to increasing the liquid pressure. These frictional forces arise from a resistance of the hydraulic fluid to flow through valves and passages in the system and drag of the seals of the piston on the sides of the piston walls.

One method of determining the total frictional force that must be overcome is to apply a measureable force to the input piston which just causes the output piston, with no load, to break loose and move. This force is known as the "breakaway force." Thus:

$$\text{Total input force} = \text{Input force creating lift pressure in fluid} + \text{breakaway force (friction)}. \quad (3)$$

Example B illustrates the use of this equation.

EXAMPLE B: FRICTION IN HYDRAULIC SYSTEMS

Given: Total force on the input piston of a hydraulic jack is 50 lb. Breakaway force is 15 lb (measured with no output load).

Find: Input force creating the lifting pressure in fluid.

Solution:

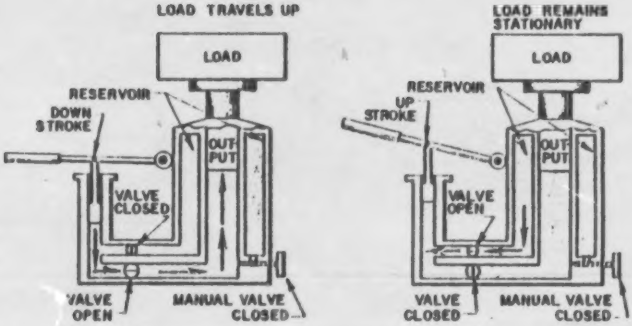
$$\text{Net input force creating pressure} = \text{total input force} - \text{breakaway force}$$

$$\text{Net force} = (50 \text{ lb}) - (15 \text{ lb})$$

$$\text{Net force} = 35 \text{ lb}.$$

Thus, even though 50 lb of force are applied, only 35 lb are applied toward lifting pressure in the jack.

In the experiment in this module, the breakaway force must be measured and used in subsequent calculations because neglecting friction seriously affects the results of the experiment.


LOAD TRAVELS UP
LOAD
RESERVOIR
DOWN STROKE
OUT PUT
VALVE CLOSED
VALVE OPEN
MANUAL VALVE CLOSED
LOAD REMAINS STATIONARY
LOAD
RESERVOIR
UP STROKE
OUT PUT
VALVE OPEN
VALVE CLOSED
MANUAL VALVE CLOSED

(a) DOWN STROKE (b) UP STROKE

Fig. 3

By observation and experience, one finds that a hydraulic jack does not produce the desired lift by a single stroke of the input piston. Also, the input piston travels a longer distance than does the output piston that supports the load. An actual hydraulic jack uses a set of two one-way valves that allow the input piston to move up and down while the output piston supporting the load only moves upward. Operation of these valves is shown in Figure 3. The one-way, "check valves" allow fluid to flow in one direction only in response to a fluid pressure difference across the valve.

To lower the load, the manual valve is gradually opened, allowing fluid to flow back into the reservoir, thereby releasing pressure under the output piston.

LABORATORY

MATERIALS

60-cm³ disposable syringe - eccentric tip without needle.
12-cm³ disposable syringe - eccentric tip without needle.
Eight, 500-g slotted weights.
Two each, 200-g, 100-g, 50-g slotted weights.
One 5-gallon bucket or plastic container.
Flexible, rubber or plastic tubing, clear 1/16-inch - inside diameter, 1/32-inch - wall thickness (medical supply house).
Clamps to secure hose to syringes.
Two ring stands and two clamps to support syringes in vertical position.
Vernier caliper.

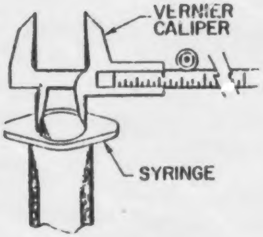


Fig. 4

PROCEDURE

Construct a simulated hydraulic jack as follows:

1. Remove plungers from both 12-cm³ and 60-cm³ syringes. Measure inside diameter of cylindrical barrels with vernier calipers as shown in Figure 4. Record these values in data table.
2. Place two syringe barrels, plungers and a twelve-inch length of tubing (unassembled) into 5-gallon container full of water.
3. Assembly of hydraulic jack simulator in this experiment is made totally under water so that no air is allowed to leak into closed system. With 60-cm³ syringe under water, push its plunger all the way to bottom to exhaust any air; fill with about 10 cm³ of water. Leave it submerged on bottom of bucket; repeat same process with 12-cm³ plunger. Connect tubing (filled with water) to eccentric tips of both plungers, while holding total apparatus under water. Remove apparatus from water; mount syringes in a vertical position, making sure that they are secure and well supported as shown in Figure 5.
4. Check operation of hydraulic system by pushing down manually on first one plunger, then other. If no air is in hydraulic system, plungers should move immediately and simultaneously.

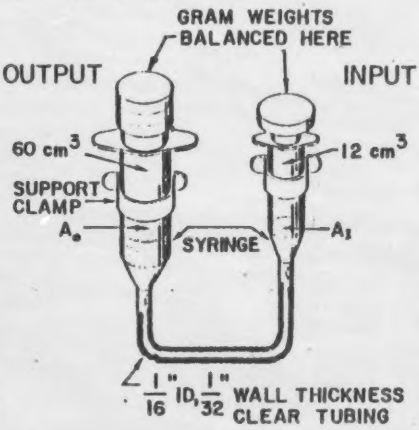

GRAM WEIGHTS BALANCED HERE
OUTPUT
INPUT
60 cm³
12 cm³
SUPPORT CLAMP
A₀
SYRINGE
Aᵢ
1"/16 ID, 1"/32 WALL THICKNESS CLEAR TUBING

Fig. 5

5. System assembled as shown in Figure 5 may be termed a hydraulic jack simulator. The 60-cm^3 syringe represents output of simulator, and 12-cm^3 syringe represents input. Manually pushing input plunger requires a certain amount of force to make output plunger move. Force required is that necessary to overcome frictional resistance of hydraulic system, that is, drag of rubber plungers, etc. To measure this breakaway force, proceed as follows. With no weights on output plunger, carefully balance slotted weight on input plunger until weight is sufficient to make both plungers begin to move and continue to move slowly. Record this weight (breakaway force) in data table. Remove weights; return fluid to input syringe (approximately 6-8 cm^3).
6. Carefully balance three 500-gram slotted masses on output plunger while manually preventing input plunger from being forced out of its tube. Place frictional force weight measured in 5 above on input syringe; carefully stack just enough additional slotted weights on top of input plunger to start system moving slowly. This shows clearly that a given force on input side causes a greater force (output weight) to be overcome. Record total force on input plunger in data table. Note: Under loaded condition of step 6, if weights are not carefully balanced and syringe barrels kept vertical, plungers rub against barrels, causing additional friction and adding to instability of apparatus. Error is thus introduced into measurements associated with experiment.
7. Repeat process outlined in steps 5 and 6 for two additional output weights; record all data in data table. Note: In this experiment masses are loaded on plungers. The fact that following ratios are equal allows use of masses and weights interchangeably:

$$\frac{\text{Output mass load}}{\text{input mass load}} = \frac{\text{output force}}{\text{input force}}.$$

SELF CHECK

1. Calculate cross-sectional area of the output and input syringes with the equation:

$$\text{Cross-sectional area} = \frac{\pi}{4} \times \text{diameter}^2.$$

Record your results in data table.

2. For each trial in the experiment, calculate difference between input mass for that trial and the input mass from trial 1 (no output load). Record these differences in data table. Gravitational force on the input mass used in trial 1 represents the amount of force required to overcome friction in the hydraulic system. This is what we called the breakaway force. Gravitational force on the additional mass loaded on the input side is used to create the pressure in the system to raise the output mass.
3. Calculate ratio of the output force (mass) to the output area; record in data table.
4. Calculate ratio of the true input force (i.e., total weight less frictional force) to the output force; record in data table.
5. Calculate ratio of the input cross-sectional area to the output cross-sectional area; compare the ratios obtained in numbers 4 and 5. Record in data table. (See Equation 2.)

DATA TABLE

Diameter of 60-cm^3 output syringe = _____ cm.

Diameter of 12-cm^3 input syringe = _____ cm.

Cross-sectional area of output (A_o) = _____ cm^2.

Cross-sectional area of input (A_I) = _____ cm^2.

Trial	Mass Loaded On Output (g)	Mass Loaded On Input (g)	Input Mass Minus Frictional Mass
1	0		0
2			
3			
4			

F_o/A_o = _____ dynes/cm^2.

F_I (true)/F_o = _____.

A_I/A_o = _____.

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REFERENCES

Beiser, Arthur. Basic Concepts of Physics. 2nd ed. Reading, MA: Addison-Wesley Publishing Company, 1972.
See especially Chapter 13.

Sullivan, J.A. Fluid Power: Theory and Applications, Section 2.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 1 F 4
MANOMETERS

FLUID APPLICATION OF FORCE

Technical Education Research Center - Southwest
TERC - SW

INTRODUCTION

The manometer measures the force of fluid pressure, and is the basic pressure-measuring instrument because of its simplicity and inherent accuracy. The actual measurement consists simply of measuring the heights of liquids in columns. The manometer is unique in being both a pressure-measuring device and a standard for calibrating other instruments. It is inherently accurate, since it contains no gears, no pressure-tight bearings, no bellows, no diaphragms, and no parts subject to wear, age or fatigue.

In this module, the student will be introduced to the basic concepts of pressure measurement with manometers. General characteristics of various U-tube manometers are described; and measurement of positive pressure, vacuum and differential pressure is illustrated. Industrial variations - the well-type manometer and the inclined manometer - also are presented. In the laboratory, the student will use a U-tube manometer to verify the calibration of a pressure gage.

PREREQUISITES

Student should have completed Concept Module I-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the terms:

  Differential pressure.
  Concave meniscus.
  Convex meniscus.
  Vacuum.

- Given the height of a manometric fluid and its density, calculate the differential pressure in the manometer.
- With the equations provided and several values for height of column level, manometer fluids, differential pressure and density values, solve typical manometer problems.

- Diagram, with suitable labeling, the following manometers:

  U-tube.
  Well-type.
  Inclined tube.
  State the advantages/disadvantages of each type.

- Convert pressure in "centimeters of mercury" or "inches of mercury" to equivalent pressure units in pounds/in^2 and vice versa.

- Given the appropriate equipment, verify the calibration for a pressure gage with a U-tube manometer.

DISCUSSION

The word "manometer" is derived from "mano" (rare or thin) and "meter" (measure) and literally means the measurement of something rare, such as air or gas. Manometers, indeed, are used in the measurement of air or gas pressure, as well as in the measurement of pressure of liquids in motion.

BASIC CONCEPTS OF PRESSURE MEASURE MEASUREMENTS

Pressure is a measure of force applied to an area, and is defined according to Equation 1.

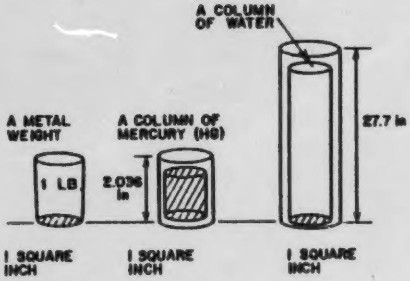
$$p = \frac{F}{A} \qquad (1)$$

where

p = pressure.
F = force.
A = area.

Some common units of pressure are pounds per square inch, pounds per square foot, newtons per square meter and dynes per square centimeter. Figure 1 illustrates how a pressure of one pound per square inch can be produced using different types of weights.

Pressure in a liquid increases with depth and is the same in all directions. A diver practicing at a deep swimming pool experiences increase in pressure in his ear drums as he dives to the bottom of the pool. In a column of fluid, such as shown in Figure 2, the pressure depends not only upon the depth, but also upon the weight density of the fluid. The difference in the pressures p_1 and p_2 shown in Figure 2 depends only upon the distance between the two points (levels) and upon the weight density of the fluid. This pressure difference is stated in Equation 2.



PRESSURE = 1 POUND PER SQ. INCH

FIG. 1

$$p_2 - p_1 = \rho h \qquad (2)$$

where

p_1 = pressure at level 1.
p_2 = pressure at level 2.
ρ = weight density of liquid.
h = height of liquid.

Equation 2 is the basic equation that governs all manometers.

Table 1 illustrates some common manometer fluids and their densities.

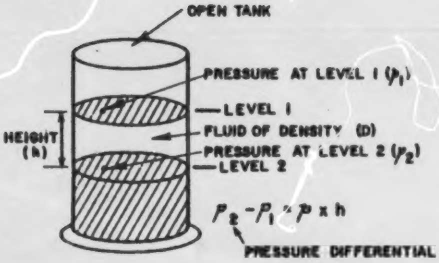

OPEN TANK
PRESSURE AT LEVEL 1 (p_1)
LEVEL 1
HEIGHT (h)
FLUID OF DENSITY (D)
PRESSURE AT LEVEL 2 (p_2)
LEVEL 2
$P_2 - P_1 = \rho \times h$
PRESSURE DIFFERENTIAL

FIG. 2

TABLE 1 WEIGHT DENSITY OF COMMON MANOMETER FLUIDS

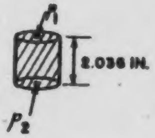
Substance	Density
Water	62.4 lb/ft^3 0.0361 lb/in^3 1 g/cm^3 (mass density)
Mercury	0.491 lb/in^3 13.60 g/cm^3 (mass) density)

Example A illustrates the use of Equation 2 and Table 1 in solving a problem.

EXAMPLE A: CALCULATION OF PRESSURE DIFFERENCE

Given: Two points (p_1 and p_2) in a column of mercury are separated by 2.036 in.

Find: Difference in pressure between p_1 and p_2.


P_1
2.036 IN.
P_2

Solution:

$$p_2 - p_1 = \rho h$$

$$= (0.491 \ \frac{\text{lb}}{\text{in}^{32}})(2.036 \text{ in})$$

(See Table 1.)

$p_2 - p_1 = 1$ lb/in^2 (Recall Figure 1.)

The metric, SI units of pressure are the Pascal and the bar. A Pascal (Pa) is a newton per square meter, a relatively small unit of pressure. Therefore, units kPa (1000 N/m^2) frequently are used. A simplifying practice in manometry is to express pressures in terms of heights of specific manometric fluid, such as "60 cm of mercury" or "20 inches of water." When expressed in this form, the addition of pressure is simplified: the heights of liquid columns simply are added. A set of unit relationships for different pressures is given in Table 2.

TABLE 2. UNIT CONVERSIONS FOR PRESSURE

1 Pa = 1 N/m^2, 1 bar = 10^5 Pa.
1 inch of water = 0.0361 lb/in^2.
1 inch of Hg = 0.491 lb/in^2.
1 mm of Hg = 0.133 kPa ≈ 1/8 kPa.
1 inch of water = 0.249 kPa ≈ 1/4 kPa.
1 inch of Hg = 3.3858 kPa ≈ 3.4 kPa.
1 psi = 6.895 kPa ≈ 7 kPa.
1 atmosphere = 14.7 lb/in^2.

AIR PRESSURE, VACUUM, ATMOSPHERIC PRESSURE

The force per unit area which is exerted upon any body immersed in air at one atmosphere is approximately 14.7 lb/in^2. One standard atmosphere is defined as follows:

1 standard atmosphere (air pressure at sea level) = 760 mm (29.92 in) of mercury (Hg) at 0°C (32°F) having a density of 13.6 g/cm^3
= 14.7 lb/in^2
= 1.013×10^5 N/m^2
= 1.013×10^6 dynes/cm^2

Barometric pressure is atmospheric pressure at a particular location.

Partial vacuum is a space in which there are very few atoms or molecules. A perfect vacuum is a theoretical space in which no atoms are present. The term, "vacuum" is generally taken to mean a space containing air or gas at well below atmospheric pressure, although, literally speaking, this space should be referred to as a partial vacuum. "Vacuum" is sometimes spoken of as "negative pressure" relative to a local barometric pressure.

Gage pressure is the difference in a pressure being read from a gage and the equivalent atmospheric pressure. Absolute pressure is the actual pressure of air or vacuum being measured. Manometers may measure gage pressure or absolute pressure, depending upon their construction. Figure 3 illustrates the relationship between the scales for pressure measurement.

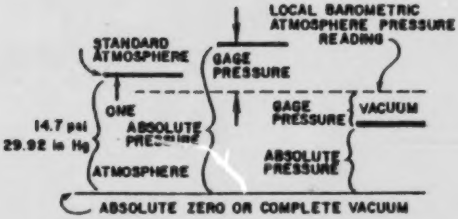
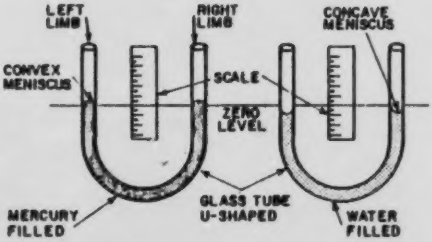

LOCAL BAROMETRIC
ATMOSPHERE PRESSURE
READING
STANDARD
ATMOSPHERE
GAGE
PRESSURE
ONE
ABSOLUTE
PRESSURE
14.7 psi
29.92 in Hg
ATMOSPHERE
GAGE
PRESSURE
VACUUM
ABSOLUTE
PRESSURE
ABSOLUTE ZERO OR COMPLETE VACUUM

FIG . 3

THE MANOMETER

The simplest reliable device for measuring moderate static pressure differences is the U-tube manometer. A simple U-tube manometer consists of a tube made of glass or other transparent material bent into the shape of a U, with both ends left open. A few spoonfuls of water poured into the tube is all that is required to make a manometer.

The U-tube generally is filled about halfway with distilled water, oil, mercury or any other liquid which flows easily, although mercury and water are most common. Figure 4 shows two liquid-filled manometers.


LEFT
LIMB
RIGHT
LIMB
CONCAVE
MENISCUS
CONVEX
MENISCUS
SCALE
ZERO
LEVEL
GLASS TUBE
U-SHAPED
MERCURY
FILLED
WATER
FILLED

(a) (b)

FIG . 4

For low pressures, the choice of liquids can be oil or water. The densities of such liquids are very low compared with mercury; therefore, the liquids are more sensitive to slight change in pressure.

Each liquid has its advantages and disadvantages as a manometric fluid. Water tends to evaporate; and if not distilled or demineralized, it tends to leave a lime deposit on the tube wall, which makes future readings difficult. Water also wets the wall of the glass tube causing the meniscus to be lower at the center than at the side, as shown in Figure 4. For higher pressure ranges, mercury is used. Clean, highly-refined mercury will not wet the walls of the tube, but will form a menisucs higher in the center than at the edges. Regardless of the shape of the meniscus, the manometry liquid levels are determined at the point at which the liquid meets the tube wall, because this point can be determined accurately by visual inspection. Consistency is all that is required, however, since pressure measurements always involve a difference in two level readings.

When no pressure is applied to either limb of a manometer, the liquid remains at zero reading on the scale. The manometer is a self-balancing instrument because it basically balances gravitational forces and applied pressures on the two limbs of the U-tube. When some outside pressure is applied to the left limb, the manometer fluid is pushed downward in the left limb and upward in the right limb until equilibrium or static conditions are achieved. This condition is illustrated in Figure 5.

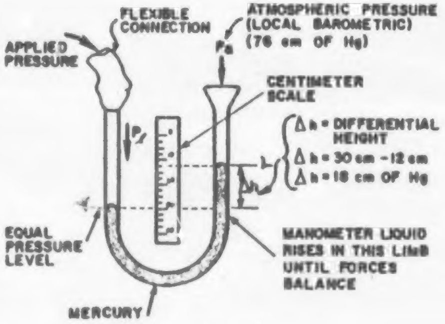

FLEXIBLE CONNECTION
ATMOSPHERIC PRESSURE (LOCAL BAROMETRIC) (76 cm OF Hg)
p_a
APPLIED PRESSURE
CENTIMETER SCALE
p_l
Δh = DIFFERENTIAL HEIGHT
Δh = 30 cm - 12 cm
Δh = 18 cm OF Hg
Δh
EQUAL PRESSURE LEVEL
MANOMETER LIQUID RISES IN THIS LIMB UNTIL FORCES BALANCE
MERCURY

FIG. 5

When the equilibrium condition shown in Figure 5 is established, pressure in the left limb is balanced by (and equal to) the total pressure in the right limb. Stated in a balanced equation form:

$$p_{left} = \Delta h + p_a \qquad (3)$$

where

p_{left} = pressure in left limb of manometer.
Δh = differential pressure height of fluid in right column.
p_a = atmospheric pressure.

Use of Equation 3 in solving a problem is shown in Example B. The simple relationships expressed by Equations 2 and 3 facilitate the use of the manometer to calibrate other types of pressure gages. If a manometer and some other pressure gage are subjected to the same applied pressure and the same atmospheric pressure, the differential height of the column of manometric fluid can be read and compared to the gage reading of the other pressure-reading device.

EXAMPLE B: CALCULATION OF PRESSURE FROM MANOMETER READING

Given: Information shown in Figure 5.

Find: (a) Applied pressure in centimeters of mercury.
(b) Applied pressure in lb/in^2.

Solution: (a) Use Equation 3:

$$p_{left} = \Delta h + p_a$$
$$= 18 \text{ cmHg} + 76 \text{ cmHg}$$
$$p_{left} = 94 \text{ cmHg}$$

(b) From Table 1, density of mercury = $0.491 \ lb/in^3$

$$p_{left} = p_2 - p_1 = \rho\Delta h \qquad \text{(Equation 2)}$$
$$= (0.491 \ lb/in^3)\left(94 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}}\right)$$
$$p_{left} = 1.82 \ lb/in^2 \text{ (absolute pressure)}$$

If the direction of an "applied" pressure is reversed, a pressure less than atmospheric is produced in the left-hand limb; i.e., a partial vacuum. This measurement is shown in Figure 6.

Pressure on the left must equal pressure on the right when static conditions are achieved. The balanced condition is illustrated by Equation 4.

$$p_{left} + \Delta h = p_{right} = p_a,$$

or

$$p_{left} = -\Delta h + p_{right} \qquad (4)$$

where

p_{right} = pressure in right limb of manometer.

Equation 4 is utilized to solve a problem in Example C.

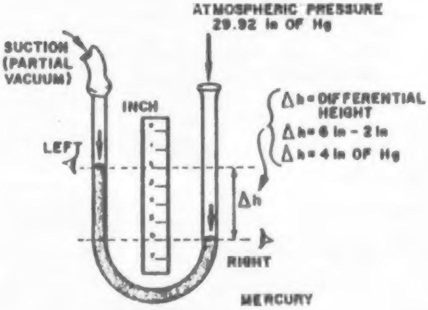

ATMOSPHERIC PRESSURE 29.92 in OF Hg
SUCTION (PARTIAL VACUUM)
INCH
Δh = DIFFERENTIAL HEIGHT
Δh = 6 in - 2 in
Δh = 4 in OF Hg
LEFT
Δh
RIGHT
MERCURY

FIG. 6

EXAMPLE C: CALCULATION OF PRESSURE FROM MANOMETER READING

Given: Information shown in Figure 6.

Find:
(a) Calculate the "vacuum" gage pressure in lb/in^2.
(b) Calculate the absolute pressure, p_{left}, in inches of Hg and in lb/in^2.

Solution: (a)

$$\text{Gage pressure} = p_{left} - p_{right}$$
$$= \rho_{Hg}h$$
$$= (0.491\ \text{lb/in}^3) \times (4\ \text{in})$$
$$\text{Gage pressure} = 1.964\ \text{lb/in}^3$$

(But note p_{left} is at a pressure below p_{right}.)

(b) Use Equation 4:

$$p_{left} = -\Delta h + p_{right}$$
$$p_{left} = -4\ \text{inches of Hg} + 29.92\ \text{inches of Hg}$$
$$= 25.92\ \text{inches of Hg}$$
$$= (25.92\ \text{in}) \times (0.491\ \text{lb/in}^3)$$
$$p_{left} = 12.7\ \text{lb/in}^2$$

(p_{right} is $14.7\ \text{lb/in}^2$.)

The U-tube manometer also is used to measure differential pressures in which the difference in the pressures of the fluids is of prime interest. Differential pressure measurement is illustrated in Figure 7.

The differential pressure measurement can be expressed in terms of $(p_1 - p_2)$ units or in terms of the differential height of manometer fluid column, Δh.

In the measurement of fluid differential pressure with a manometer, two immiscible fluids, each of different density, are present in the manometer tubes (Figure 8). As before, the total pressure on the left-hand side must balance the total pressure on the right-hand side in a static situation. The differential pressure which results is given by Equation 5:

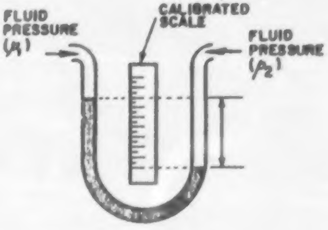


FIG. 7

$$p_1 - p_2 = \Delta h \, (\rho_H - \rho_L) \qquad (5)$$

where

ρ_H = weight density of 1st liquid (higher).
ρ_L = weight density of 2nd liquid (lower).

Equation 5 is used to solve a problem in Example D. The difference in densities of the two liquids is known as the wet-leg correction.

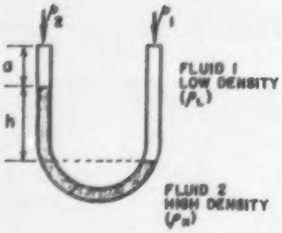


FIG. 8

EXAMPLE D: CALCULATION OF PRESSURE DIFFERENTIAL

Given: A U-tube manometer filled with mercury is used to measure the differential pressure across a venturi tube in a water line. While a steady water flow is occurring, a height, Δh, is measured to be 10 inches.

Find: The differential pressure ($p_1 - p_2$) in lb/in^2.

Solution: From Table 1:

Mercury density (ρ_H) = 0.491 lb/in^3

and

Water density (ρ_L) = 0.0361 lb/in^3

Therefore:

$$\begin{aligned} p_1 - p_2 &= \Delta h \, (\rho_H - \rho_L) \\ &= (10 \text{ in})(0.491 \text{ lb/in}^3 = 0.0361 \text{ lb/in}^3) \\ &= (10 \text{ in})(0.455 \text{ lb/in}^3) \end{aligned}$$

$$p_1 - p_2 = 4.55 \text{ lb/in}^2$$

Notice that if the wet-leg correction had not been made, the pressure obtained would have been 4.91 lb/in^2 (uncorrected).

INDUSTRIAL VARIATIONS OF THE MANOMETER

WELL-TYPE MANOMETERS

U-tube manometers require that two readings be taken and then added algebraically before the desired final pressure can be obtained. To simplify this procedure, a well, or reservoir, is substituted for one of the columns of the U-tube. The liquid level in the reservoir has a much larger surface area, as shown in Figure 9. Pressure applied to the reservoir forces the level down slightly while the fluid enters the measuring column. If the diameter of the reservoir is much learger than the diameter of the measuring tube (in commercial models, a ratio of about 250 to 1), then the drop in the reservoir level is considered insignificant; and only the level in the measuring tube need be read to determine Δh directly from the calibrated scale. The pressure differential is given by Equation 6.

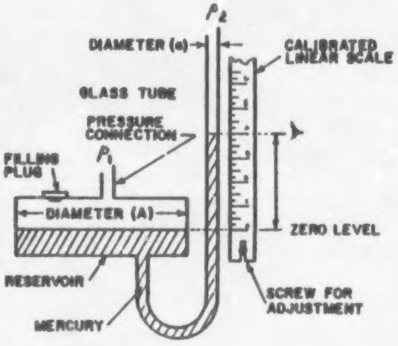


FIG. 9

$$p_1 - p_2 = \rho_{Hg}\, h \qquad (6)$$

(if a << A).

In actual practice, the well manometer should be zero adjusted before readings are taken. It also should be leveled by an attached level gage.

THE INCLINED-TUBE MANOMETER

Extremely small differential pressures frequently are measured with a special U-tube manometer. One tube is inclined at an angle (θ) to the horizontal and to the vertical tube (that consists of a relatively large well), as shown in Figure 10. The main purpose of the inclined tube is to achieve a longer scale than possible with the ordinary vertical U-tube manometer for the same pressure differential.

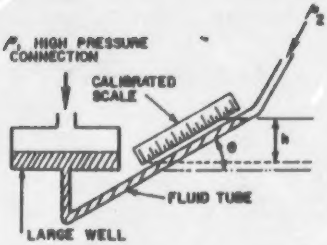


FIG. 10

For example, if the tube is inclined to the horizontal at some angle in such a manner that 20 units measured along the tube equals 1 unit vertical rise, then a rise, h, in the level of the liquid in the tube will produce a movement of the liquid along the tube 20 times as great. Therefore, the movement for a small change in level is more readily detected than in a vertical manometer. The increased sensitivity of an inclined manometer can be illustrated by a six-inch vertical range, expanded by an inclined manometer to cover a scale 48 inches in length. An instrument of this type easily could be graduated into hundredths of an inch. With water as the indicating manometer fluid, a tenth of an inch on the scale would correspond to a pressure differential of approximately 0.0005 psi.

The pressure differential measured by an inclined manometer is given by Equation 7.

$$p_1 - p_2 = \rho d F \qquad (7)$$

where

d = inclined scale reading.
F = scale factor (vertical height/unit scale length), sin θ.

The inclined manometer is used mostly for low-pressure differential measurements of boiler draft, furnace pressure, static pressures in air-conditioning systems, and for determining differential pressures across air filters and fans.

In this module, the method of reading the manometer has been limited to visual inspection and direct reading of a scale. Modern industry, however, has developed a variety of more sophisticated and accurate methods for following the rise and fall of a manometer fluid in a column.

SUMMARY

The manometer measures the force of fluid pressure. Because of its simplicity and accuracy, the manometer is the basic pressure-measuring device. It can measure positive pressure, vacuum and differential pressure, and can be used to calibrate other instruments. The inclined-tube manometer is an extremely accurate and useful design for industrial uses.

LABORATORY

MANOMETRIC VERIFICATION OF PRESSURE-GAGE CALIBRATION

MATERIALS

Manometer (vacuum and pressure) - A U-shaped glass tube with arms one meter in length, both ends open, mounted on meter-stick scale clamped to vertical support. (See Figure 11.)
About 175 g (6 oz) of mercury.
Thistle tube for filling manometer with mercury.
Vacuum pump, hand-operated, high-impact polystyrene plastic, with attached pressure gage calibrated in centimeters and inches. Pressures up to 7 inches positive and 25 inches (vacuum) are capable. Pumping rate is 15 cm^3 per stroke (Sargent-Welch Scientific Company). (See Figure 11.)
Two-foot length of transparent vinyl plastic tubing 1/4" inside diameter (connection from pump to manometer).

PROCEDURE

1. Arrange equipment as shown in Figure 11.

2. Attach connection tube to positive pressure outlet of hand pump, and gently squeeze handle until needle on pressure gage registers one centimeter of Hg. Hold this pressure while reading column level scale readings. Record each scale reading in the Data Table.

3. Increase the pressure in approximately one centimeter steps, as indicated on the pressure gage, holding pressure while reading column heights. Record each column level scale reading for each pressure in Data Table until the maximum calibrated pressure on the gage is reached.

4. Carefully and slowly release pressure on manometer. Change connection tube to vacuum outlet of hand pump.

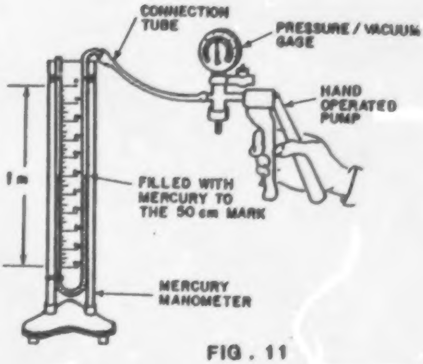


FIG. 11

5. Gently squeeze handle of pump to produce a one-centimeter "vacuum" on pressure gage. Hold this "vacuum," and record manometer levels in Data Table. Repeat this procedure until maximum "vacuum" for which the pressure gage is calibrated is reached.

6. Carefully release "vacuum."

7. From observations recorded in Data Table, calculate manometer height for each pressure reading by subtracting level of right limb from level of left limb. Record results in Data Table. Do these results differ from those indicated on the pressure-gage dial?

8. With Equation 2, calculate the pressures in lb/in^2 for each reading. Record results in Data Table.

9. Suggest a method by which the pressure gage could be recalibrated in lb/in^2 without breaking into the body of the gage.

DATA TABLE

Pressure Gage Reading cm of Hg	Manometer Left Level cm of Hg	Manometer Right Level cm of Hg	Manometer Pressure (1) cm of Hg	Pressure (2) lb/in^2
Vacuum				

Notes: (1) Manometer pressure = (left level) - (right level).
(2) Pressure = (0.491 lb/in^3) x manometer height.

SELF CHECK

1. Define the terms:

    Differential pressure.
    Concave meniscus.
    Convex meniscus.
    Vacuum/partial vacuum.

2. The differential height of a mercury column in a barometer is 76 cm. Calculate the differential height in meters, if oil (density 0.90 g/cm^3) is used instead of mercury (density 13.6 g/cm^3).

3. If a mercury manometer is used to measure a differential pressure of 2.0 psi, calculate the differential height between the two levels of mercury columns in inches.

4. In question 4 above, what would be the difference in column levels if water were used instead of mercury.

5. A pressure was applied to one arm of an open tube manometer (mercury filled) and the level difference was found to be 12.0 inches. Compute the applied pressure in pounds per square inch.

6. Sketch a diagram of a U-tube manometer, and label its parts.

7. Sketch a diagram of a well-manometer labeling its principal parts. State its advantages over the U-tube manometer.

8. Sketch a diagram of an inclined-tube manometer. State its advantages over the U-tube manometer.

9. For problems 3, 4 and 6, convert the results to metric and/or Enlish systems units.

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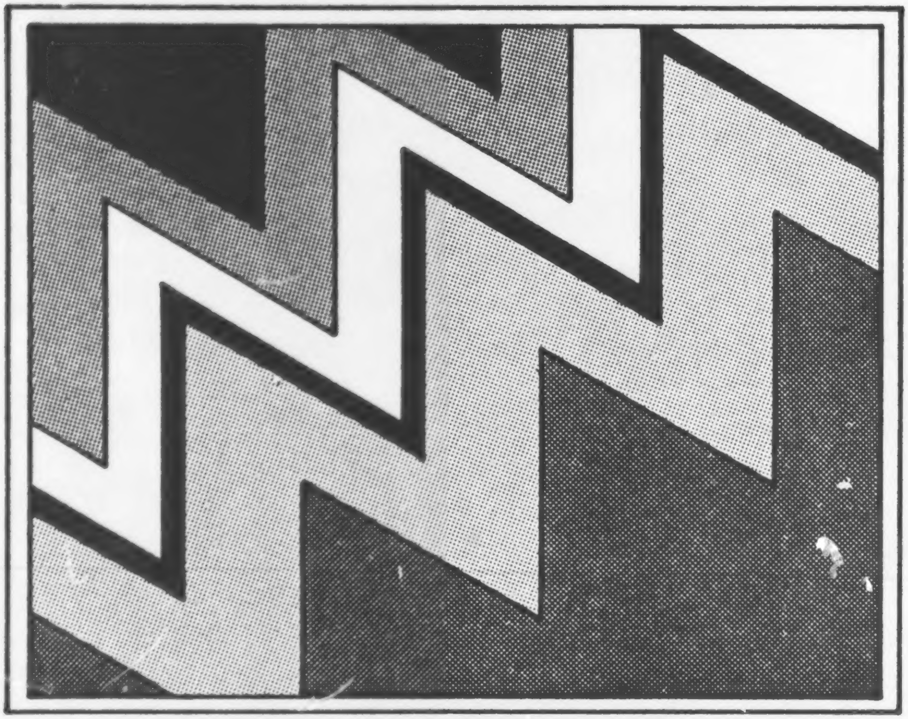
UNIFIED TECHNICAL CONCEPTS

MODULE 1 E 1
POTENTIAL DIFFERENCE AND CHANGE FLOW



ELECTRICAL APPLICATION OF FORCE

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

To understand the nature of electricity, one must know the nature of the forcelike quantity behind it. This forcelike quantity is called "potential difference," although in different contexts it also is commonly referred to as "voltage," "electromotive force," "emf," and "electrical pressure."

Potential difference (voltage) is the force that moves electrical charges in a circuit to produce current. It is the forcelike quantity behind light produced in lamps, turning of motors in industry, current through an automobile ignition, and heat generated by a toaster.

In this module, the student will learn the sources of potential difference, its characteristics, and the way in which it is measured. In the laboratory, the student will measure the voltage in electrical circuits.

PREREQUISITES

Student should have completed Concept Module I-0, "Force."

OBJECTIVES

To complete this module, the student should be able to:

- Define the following terms:

  Electron.
  Potential difference.
  Volt.
  Coulomb.
  Positively charged matter.
  Negatively charged matter.

- Name and give an example of five ways in which a potential difference (voltage) can be produced.
- Describe in a short paragraph how charge is transferred when a potential difference is present across a wire.

- Define in a short paragraph the meaning of the "reference point" in an electric circuit and how it is used in describing voltages.
- Given the appropriate equipment, measure voltage in electrical circuits.

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DISCUSSION

Concept Module 1-0, "Force," stated that all matter is made of atoms having positively-charged cores surrounded by orbits of negatively-charged particles called electrons (Figure 1).

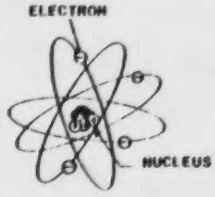


FIG. 1

Any atom in its natural state has a new neutral charge because positive charges equal negative charges. However, in some atoms, such as those in copper, electrons are so loosely bound to the parent atom that a small amount of energy can strip them away. When this happens, the parent atom becomes a positively-charged ion, and the electron(s) becomes a free negative charge. By definition, something neutral becomes "positively charged" when it has a shortage of electrons; and something neutral becomes "negatively charged" when it has an excess of electrons.

Figure 2 shows the forces between two charged bodies. If the bodies have like charges, either positive or negative, each experiences an electrical force of repulsion directed away from the other body. If the bodies have unlike charges (one positive and one negative), the electrical forces are those of attraction that draw the bodies toward each other. These electrical forces of attraction and repulsion give rise to the forcelike quantity known as "voltage," or "potential difference" in electrical systems.

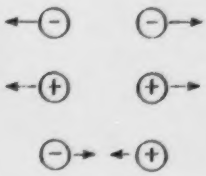


FIG. 2

A potential difference exists whenever positive and negative charges are not evenly distributed in a system. Figure 3 shows the basic components of a battery. Electrolite is a solution that transports electrons from the positive electrode (anode) to the negative electrode (cathode) by chemical action. This transfer results in an excess of electrons on the cathode and a shortage of electrons on the anode. The forces of repulsion among the electrons on the cathode tend to force electrons away from the cathode. Since the anode is positively charged, it pulls electrons toward it. Thus, if electrons are free to move, they travel from the cathode to the anode. The potential difference, measured in volts (V), is proportional to the force on an electron.

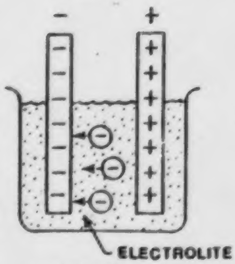


FIG. 3

OTHER SOURCES OF POTENTIAL DIFFERENCE

FRICTION

Static electricity is a common occurrence, particularly during a dry, cold winter. When certain dissimilar materials, such as wool, hard rubber, or glass and fur are rubbed together, one material will strip electrons away from the other. If enough potential difference is present, sparks can be seen jumping between the two materials.

The sparks are caused by an instantaneous current of electrons from the negatively-charged material to the positively-charged material.

GENERATOR

In a generator, conductors in the rotating armature, or coil, are kept moving in a magnetic field provided by the field assembly of the generator or by a permanent magnet, causing current to flow in the armature conductors. The two ends of the armature coil wire will have a potential difference between them.

THERMOCOUPLE

Heat also can be used to produce a voltage, as in the thermocouple. A thermocouple consists of two dissimilar metals connected together at both ends, called "junctions." When heat is applied to one junction, electrons move from one metal to the other, and a potential difference is produced.

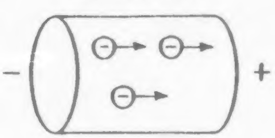
PHOTOELECTRIC

When light beams strike a photosensitive material, electrons in the material are displaced, and a difference of potential is produced. This phenomenon is used in such devices as light meters, photo tubes, and solar cells.

CHARGE FLOW

After the charges are separated, there is a strong tendency for them to reunite. When an electrical path is provided between the negatively-charged region and the positively-charged region, electric charge will flow until the potential difference is zero. If potential difference can be maintained, as in a battery or in a generator, a continuous current exists in the complete circuit.

Charge can be transferred by the movement of either positive or negative charges. When a current flows through a wire, charge is transferred by moving electrons.



WIRE SEGMENT

FIG. 4

In metals, the outer electrons are bound so loosely to their atoms that very little energy is necessary to remove them. These electrons drift through the metal at random, moving from atom to atom. When a potential difference is applied across the metal, the electrical forces cause all the electrons to move away from the cathode and toward the anode. Excess electrons from the cathode of the source enter the negative end of the wire, and electrons at the positive end leave the wire for the more positive anode. The result is a charge transfer that will eventually return the system to electrical equilibrium.

Electrical charge is measured in coulombs. One coulomb is the charge of 6.25×10^{18} electrons. By definition, one volt is the potential difference between two points in an electrical circuit when one joule of energy is released as one coulomb of charge is transferred between the two points. Thus, a volt is equivalent to a joule per coulomb.

VOLTAGE MEASUREMENT

A voltmeter is a measuring instrument used to determine the potential difference between two points. The potential difference causes a small current to flow through the voltmeter. This current produces a deflection of the needle of the meter. Since needle deflection is proportional to current and current is proportional to applied voltage, the meter can be calibrated to display the potential difference directly in volts.

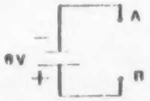


FIG. 5

A voltage always is the difference in electrical potential between two points. One of these points is chosen as the reference point, and the potential of the other point is measured with respect to that reference. If the second point is positive with respect to the reference point, the voltage is positive. If the second point is negative with respect to the reference, the voltage is negative. In Figure 5, the voltage "from A (reference) to B" is 6V; the voltage from B to A is -6V.

The voltage, or difference of potential, across a battery is measured by connecting a voltmeter as illustrated in Figure 6.

Notice that the common (negative) meter lead is connected to the negative terminal and that the positive lead is connected to the positive battery terminal.

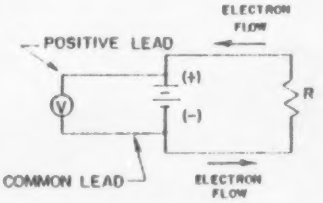


FIG. 6

When using a voltmeter or a VOM, observe the following steps:

1. Set meter on positive or negative volts d.c. (or on a.c. as required).

2. First set range switch for maximum range that can be expected. Meter range switch then can be moved to a lower range as needed. Never start with too small a range because meter will be "pegged," and the meter movement may be damaged permanently.

3. Connect common lead of meter to desired point of zero reference.

4. Connect positive probe to desired voltage point.

5. With power applied, meter can be read. Carefully interpret selected scale, and read meter.

SUMMARY

All matter is composed of atoms containing positive and negative electrical charges that obey these rules: (1) Like charges repell, and unlike charges attract. (2) When these charges are separated to produce an excess of electrons (negative charges) in one area and an excess of positive charges (a lack of electrons) in another area, a potential difference exists. (3) If these two areas are connected by an electrical conductor, current will flow until the system reaches equilibrium.

A voltmeter is a device used to measure the potential difference between two points in an electrical circuit. The unit of potential difference is the volt, which is equal to one joule per coulomb (J/C).

VOLTAGE MEASUREMENTS

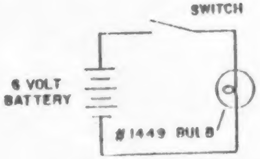


FIG. 7

MATERIALS

Several copper wires or leads.
One 6-V dry cell battery (or power supply).
One voltmeter or volt-ohmmeter (VOM).
Two 10-kΩ, 1/4-watt resistors.
One single-pole, single-throw toggle switch.
One No. 1449 bulb and socket.

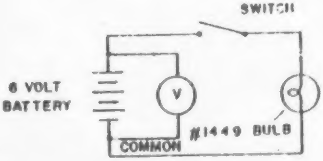


FIG. 8

PROCEDURE

1. Connect circuit as illustrated in Figure 7. Do not connect voltmeter at this time.

2. Close switch. Does lamp glow? ________.

3. Connect voltmeter (or VOM) as shown in Figure 8. What is voltage reading? ________.

4. Connect voltmeter as shown in Figure 9. What is voltage reading? ________.

5. Connect circuit as shown in Figure 10.

6. With common lead of voltmeter at point B, measure voltage across R_1 by placing positive probe at point A. What is voltage across R_1? ________.

7. With common lead at point C, place positive probe at point B. What is voltage across R_2? ________.

8. Figure 11 shows same circuit, but with point C grounded. This means that point C is reference point ("0" volts point) for rest of circuit.

9. Since point C now is the "0" volt point, measure voltage at points A and B with respect to point C by placing common lead at point C and probe lead on points B and A, in turn. Point B should read 3 volts with respect to point C. Does it? ________. Point A should read 6 volts with respect to point C. Does it? ________.

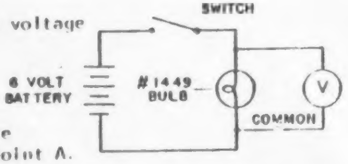


FIG. 9

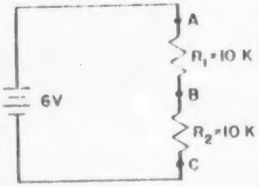


FIG. 10

10. Refer to Figure 12. Notice that point B now is used as grounded reference point. Measure voltage at points A and C first by placing common lead on B and then, in turn, by placing probe lead on points A and C. Measure point A with respect to point B. V_{AB} = ______ volts. Measure point C with respect to point B. Since common is on positive side of R_2, be sure to change voltmeter to read negative d.c. volts. V_{BC} = ______ volts.

11. Measure points B and C with respect to point A (A grounded) as shown in Figure 13. Place common lead at point A and probe lead at points B and C, in turn. Be sure to change voltmeter to read negative volts, since point A, zero reference, is at a higher potential than either point B or C.

V_{AB} = ______ volts.
V_{BC} = ______ volts.

12. Why is voltage often called "potential difference"?

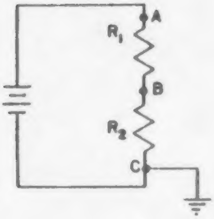


FIG. 11

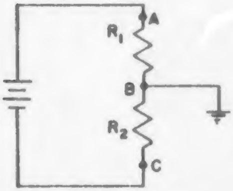


FIG. 12

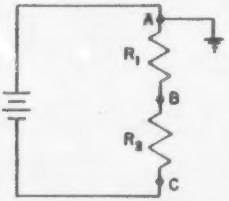


FIG. 13

SELF CHECK

1. Define these terms:

Electron.
Potential difference.
Volt.
Coulomb.
Positively-charged matter.
Negatively-charged matter.

2. Write a paragraph summarizing correct use of the voltmeter (or VOM) to measure voltage.

3. Name five sources of voltage; give an example of each.

4. Describe in a short paragraph how charge is transferred through a wire.

5. Explain the importance of the reference point in measurement of voltage in a circuit.

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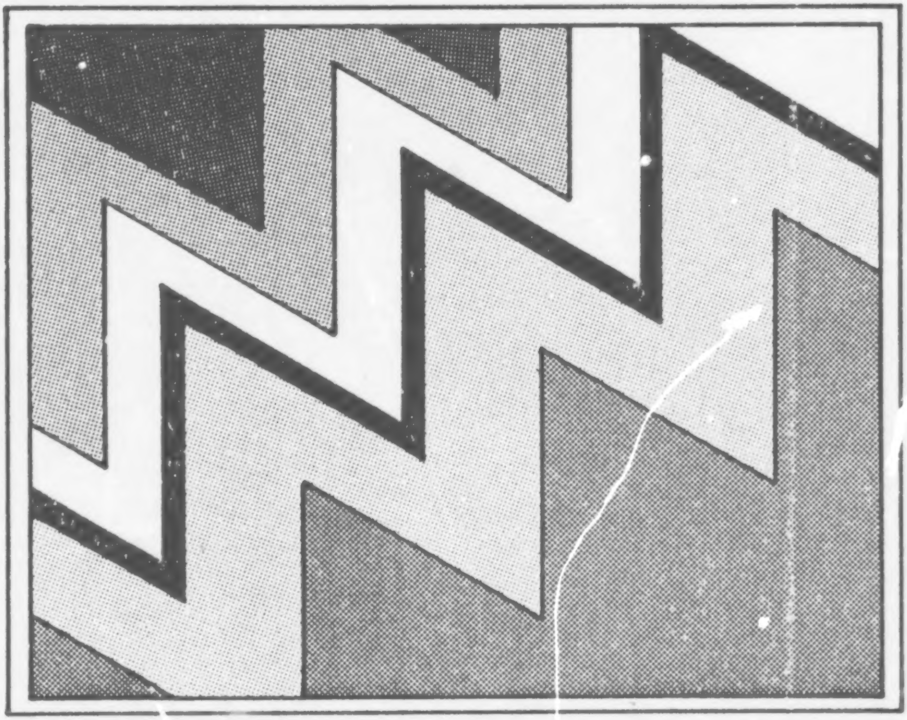
UNIFIED TECHNICAL CONCEPTS

MODULE 1 E 2

SERIES AND PARALLEL VOLTAGE SOURCES



ELECTRICAL APPLICATION OF FORCE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

The concept of electromotive force (emf) was introduced in Module I-0, "Force." To understand practical applications of this forcelike quantity, it is necessary to understand how several sources of emf interact in a simple circuit.

In this module, the student will examine methods of determining resultant emf when several voltage sources are present in the same circuit. Results of such calculations will be verified in the laboratory.

PREREQUISITES

Student should have completed Concept Module I-0, "Force," and Application Module IEI, "Potential Difference and Charge Flow."

OBJECTIVES

To complete this module, the student should be able to:

- Given simple circuits, calculate total emf when several sources of emf are connected in series or in parallel.
- Determine total emf of a series-parallel combination of several sources of emf.
- Specify when it is desirable to connect cells in series and when it is desirable to connect them in parallel; explain why.

DISCUSSION

As discussed in Module 1-0, "Force," any device that transforms energy from a nonelectrical form to electrical energy produces an electromotive force (emf). This emf establishes the difference in potential that causes current to flow in electrical conductors. In this module, consideration of how several sources of emf may be connected in a simple circuit to produce a total emf of possibly different values will be made. All emf's considered will be constant — unvarying with time — such as a d.c. battery. While we are not yet ready to discuss electrical resistance as such, the basic concept of resistance is important. It is sufficient to note here that resistance in an electrical circuit is the quantity that limits the current flow; the greater the resistance the lower the current with a fixed emf.

Figure 1 shows a source of emf in a circuit. The symbol in the dotted circle actually represents a battery, but it is sometimes used to represent other sources of constant emf. In this circuit the battery causes a current I to flow through the resistor R. Voltage (potential difference) between points a and b is written as V_{ab} and is equal to the emf of the battery if the battery has no internal resistance. Although emf is not really a vector quantity, it is convenient to assign it a direction of "sense." The arrow in Figure 1 indicates direction of the emf within its source. The direction of an emf is arbitrarily defined to be from the negative terminal toward the positive terminal in the source itself.

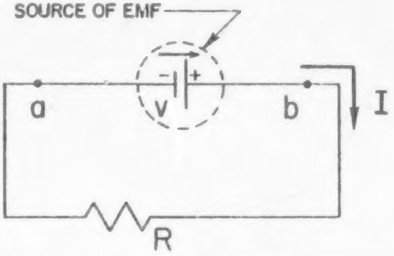


Fig. 1

Figure 2 shows two sources of emf connected in series so that their directions are the same. The potential difference between points a and b is the sum of the emf's between them (so long as no current is flowing or the sources have no internal resistance). The result is given in Equation 1.

$$V_{ab} = V_1 + V_2. \qquad (1)$$

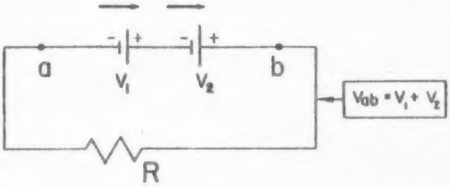


Fig. 2

Because, in traveling from point a through the batteries to point b, movement is in the same direction as the sense of both sources of emf, all quantities are positive and add as indicated in Equation 1.

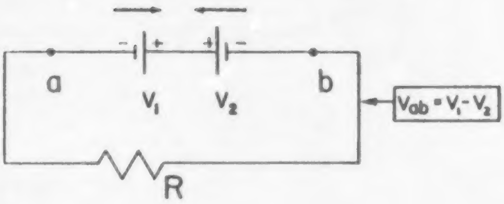


Fig. 3

Figure 3 shows a slightly different case of two sources of emf connected in series with opposing directions. In this case, voltage from point a to point b is:

$$V_{ab} = V_1 - V_2. \tag{2}$$

Now V_2 is negative because its sense is opposing the "direction of travel," defined to be from a to b.

Note that V_{ba}, which designates voltage from point b to point a (Figure 3), will be the sum of a positive V_2 and a negative V_1 so that $V_{ba} = -V_{ab}$.

Similarly, in Figure 2 the following is shown:

$$\begin{aligned} V_{ba} &= -V_{ab} \\ &= -(V_1 + V_2) \\ V_{ba} &= -V_1 - V_2. \end{aligned} \tag{3}$$

Both emf's appear as negative quantities in the equation because the sense of each opposes the new direction of potential difference measurement from b to a.

A general rule for the addition of electromotive forces in series is stated in Equation 4.

Potential difference between two points in a series connection = algebraic sum of all electromotive forces between the 2 points.

$$\text{In symbols:}\quad V_T = \Sigma\text{emf}. \tag{4}$$

In this equation, it is assumed that there are no resistances between points a and b, either internally in the batteries or between the batteries, and that the senses of all emf's are used to determine their signs in the equation.

Example A shows how to use Equation 4 in solving a problem of multiple sources of emf connected in series.

EXAMPLE A: CALCULATING POTENTIAL DIFFERENCE

Given: Illustration at right.

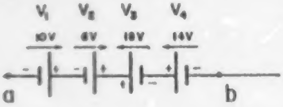
Find: V_{ab}.

Solution: Use Equation 4.

$V_T = \Sigma\ (V_1 + V_2 + V_3 + V_4)$

$V_{ab} = 10\ V + 6\ V - 18\ V - 14\ V$

$V_{ab} = -16\ V.$



V_3 and V_4 are negative because their senses oppose the direction of a to b.

Figure 4 shows two sources of emf connected in parallel with like poles together. When cells or batteries are connected in parallel, they should be connected with like poles together and should have the same emf's. Otherwise, there will be currents circulating in the local branches formed by the cells. These currents will not contribute to the current in the external resistor and will serve only to drain the cells. Note that connecting like poles together is very important when connecting sources of emf in parallel arrangements. For example, do not arrange dry cells or batteries in parallel with unlike poles together.

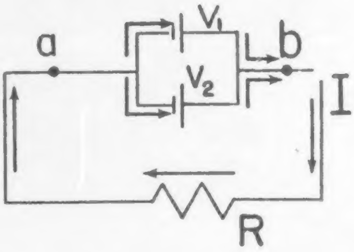


Fig. 4

From the above discussion, a general rule is derived for the addition of equal sources of emf in parallel as illustrated in Figure 4. It is given in Equation 5.

Potential difference across a parallel connection of sources of emf = emf of one of the sources. (All emf's must be equal.)

In symbols: $V_T = V_1 = V_2 = V_3 = \ldots$ (5)

Example B shows how to use Equation 5 in solving a problem that involves a parallel nest of six-volt batteries.

EXAMPLE B: CALCULATING SYSTEM VOLTAGE

Given: Three six-volt batteries connected in parallel with like poles together.

Find: Resulting total voltage across the combination, that is, such as across points a and b in Figure 4.

Solution:

$V_{ab} = V_1 = V_2 = V_3$.

$V_{ab} = 6$ V (from Equation 5).

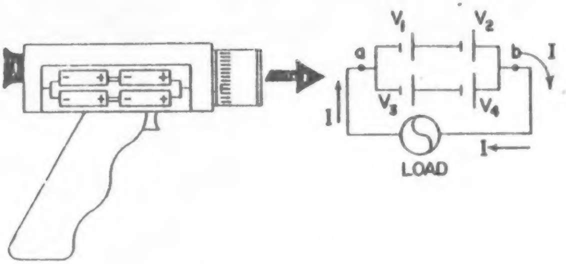


Figure 5 shows a typical application of the principles involved where there are two parallel circuits, each circuit having two series of batteries. Example C is a problem involving Figure 5.

Fig. 5

EXAMPLE C: CALCULATING MOTOR VOLTAGE

Given: Each of the batteries in Figure 5 has an emf of 1.5 volts.

Find: Voltage delivered to the camera motor.

Solution: The two series connections give:

$V_1 + V_2 = 3.0$ V (upper branch)

$V_3 + V_4 = 3.0$ V (lower branch).

These are connected in parallel for:

$V_{ab} = 3.0$ V.

Thus, 3.0 volts are delivered to the camera motor.

Summarizing the series and parallel connection of sources of emf as follows. When connected in series, their emf's add but the current through each is the same; when connected in parallel, their currents add but the emf is the same as the emf of any one of the identical sources. Cells are connected in series when it is desirable to have a given current in a comparatively high resistance. Cells are connected in parallel when large currents are to be maintained in a low resistance. For example, if a current of 0.20 amperes is to be maintained in a resistor of 50 ohms, using cells each of emf 2 volts and negligible internal resistance, five of the two-volt cells in series would provide the ten volts required. On the other hand, a current of 20 amperes is to be produced in a silver-plating cell that has a resistance of 0.1 ohm. Ohm's law states that the required voltage is 20 amps x 0.1 ohm, or 2 volts. Hence one 2-volt cell could be used. But one 2-volt cell would not be able to carry 20 amps for long without damage. It would be better, therefore, to use four 2-volt cells in parallel. In this case, the voltage supplied would still be 2 volts, but now each cell would deliver 5 amps, a current that could reasonably be carried by one cell.

Note that, if a maximum current is to be maintained in a given external resistance (load) by a given number of cells, the cells should be arranged in such a manner that their total internal resistance will most nearly equal the external load resistance. This may require a series-parallel arrangement as shown in Figure 5.

LABORATORY

MATERIALS

Two D cell batteries.
Two battery holders.
Clip leads.
Volt ohmmeter.
Three-volt bulb.
Knife switch.

PROCEDURE

1. Measure emf of each battery under "zero" current conditions; record values in data table.
2. Connect circuit as shown in Figure 6.
3. Close switch; observe light bulb.
4. Measure voltage V_{ab}; record in data table.
5. In same way, connect each circuit (B, C, D and E) shown in Figure 7; take necessary data to complete table.

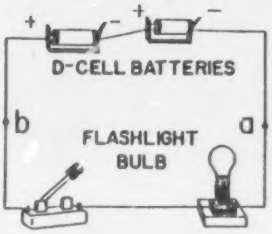
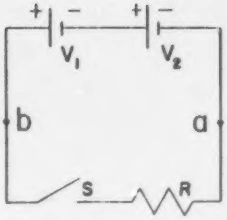
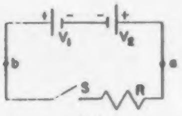

+
-
+
-
D-CELL BATTERIES
b
a
FLASHLIGHT BULB

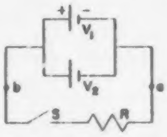
Fig. 6



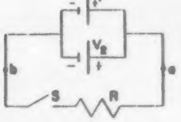
(A)



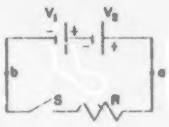
(B)



(C)



(D)



(E)

DATA TABLE

Record measured voltage value:

V_1 = __________ (emf)

V_2 = __________ (emf)

V_{ab} (A) = __________

V_{ab} (B) = __________

V_{ab} (C) = __________

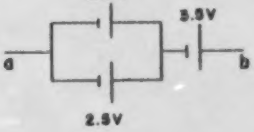
V_{ab} (D) = __________

V_{ab} (E) = __________

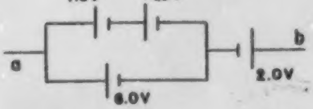
	Bulb Condition		
	Out	Dim	Bright
A			
B			
C			
D			
E			

SELF CHECK

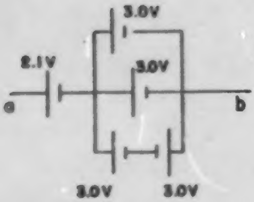
1. Examine each of the four voltage values V_{ab} for circuits A, B, C, D and E in the data table. Why might V_{ab} for circuit B and circuit D not agree with results predicted by Equations 4 and 5 respectively?
2. Explain what happens when two emf's are connected in parallel with unlike poles connected. (Remember that this is not to be done.)
3. Explain what happens when two voltage sources with unequal emf's are connected in parallel with like poles connected. (Remember that this is a wasteful arrangement!)
4. In each of the following circuits, determine voltage across the connected emf's. Indicate those circuits containing improper connections with an X.



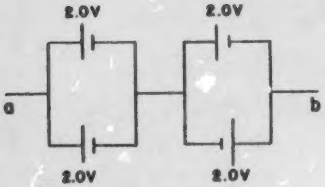
(A)



(B)


3.0V
2.1V
3.0V
a
b
3.0V
3.0V

(C)


2.0V
2.0V
a
b
2.0V
2.0V

(D)

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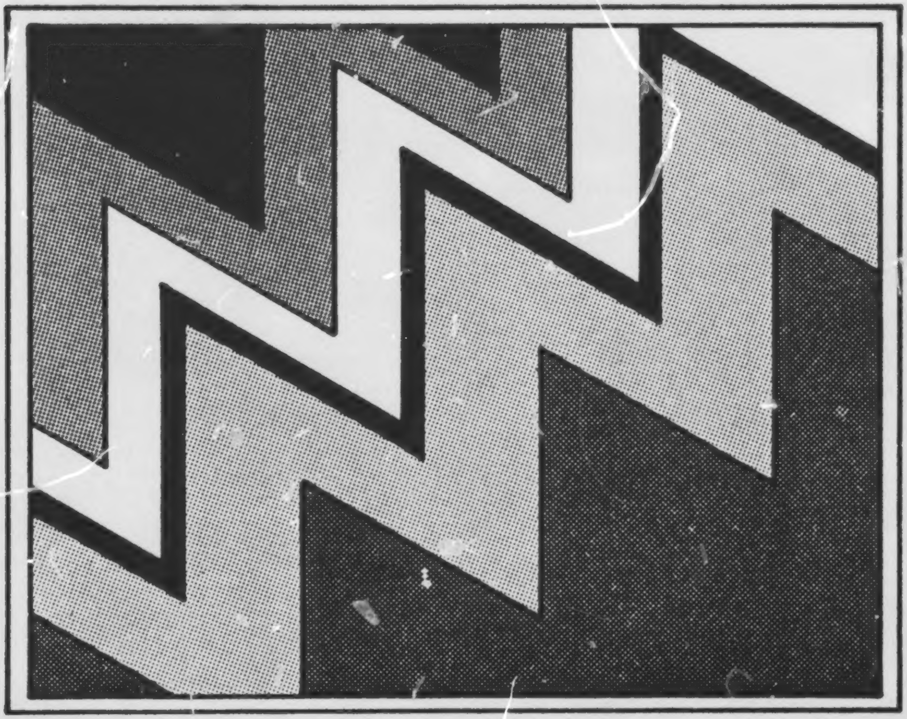
Sears, F.W.; Zemansky, M.W. and Young, H.D. University Physics. 5th ed. Reading, MA: Addison-Wesley Publishing Company, 1977.

Part II only.

UTC
UNIFIED TECHNICAL CONCEPTS

MODULE 1 E 3
ELECTRIC FORCES AND FIELDS



ELECTRICAL APPLICATION OF FORCE

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

This module introduces electric forces and electric fields encountered in the electromagnetic energy system. Electric forces exist between electrically-charged particles in much the same way that gravitational forces exist between different masses. Electric forces and electric fields are important factors in the operation of electrical current in conductors, electrical discharge, breakdown voltage, capacitors, particle accelerators, resistance-capacitance d.c./a.c. circuits and in the field of electronics in general.

This module presents the calculation and graphical representation of electric forces between particles and the electric fields around capacitors. The relationships between charge, voltage, and capacitance also is presented. In the laboratory, the student will produce and describe line patterns of electric fields.

PREREQUISITES

Student should have completed Concept Module I-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Name the two different types of electric charge and give an example of a particle with each type of charge.
- Determine the electric force between charged particles of known charge and separation.
- Draw electric field lines around point charges and charged conducting plates, and describe how field lines indicate direction and charging strength of the electric force on charged particles located in the field.
- Draw a parallel-plate capacitor including electric field lines; and describe the relationships between plate charge,

potential difference and capacitance and potential difference, plate separation and electric field strength.

- Describe the use of parallel plate capacitors and the role of the electric field in television tubes and electrostatic precipitators.
- Given the appropriate equipment, map the electric field between simple conductors carrying opposite charge.

DISCUSSION

Electric force is a simple push or pull which has its origin in electrical charge. There are two fundamental kinds of electrical charge, positive and negative. The proton is an example of positive charge; the electron is an example of negative charge. Several commonly-known particles with their appropriate electric charge (coulombs) are tabulated in Table 1.

TABLE 1. SOME ELEMENTARY CHARGED PARTICLES

Particle	Charge (coulombs)	Mass (kilograms)
Electron	-1.602×10^{-19} C	9.108×10^{-31} kg
Proton	$+1.602 \times 10^{-19}$ C	1.672×10^{-27} kg
Alpha	$+3.204 \times 10^{-19}$ C	6.648×10^{-27} kg
Neutron	No Charge	1.675×10^{-27} kg

The electron and the proton have the same charge, although the proton is about 1836 times heavier. The alpha particle has twice the charge of the proton and is about 4 times heavier. The neutron has no charge; it is an electrically neutral particle. Table 1 also shows that all the charged particles listed are very light (almost massless) and carry very little charge. For example, about 6.24×10^{18} electrons (6,240,000,000,000,000,000 electrons) are required to equal one coulomb of electrical charge; therefore, the charge on an electron or proton, the smallest charge known, is a very small part of one coulomb of charge.

FORCES BETWEEN ELECTRIC CHARGES

Different masses, such as the sun and the earth, only can attract, never repel, each other. Thus, gravitational forces - the forces between different masses - are attractive. Electrical charges, on the other hand, both attract and repel one another. Figure 1 shows that "like-charges" repel and "unlike-charges" attract. A positive charge repels another positive charge, but attracts a negative charge. Similarly, a negative charge repels another negative charge, but attracts a positive charge. The magnitude of the force between charged particles is given by COULOMB'S FORCE LAW, as

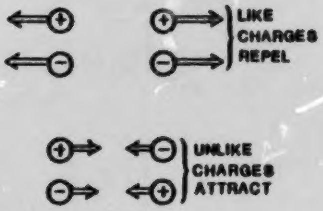


FIG. 1

indicated by Equation 1.

$$F = \frac{kQ_1Q_2}{d^2} \qquad (1)$$

where

F = the attractive or repulsive force in newtons.
k = a constant equal to $9 \times 10^9 \, \frac{\text{newton meter}^2}{\text{coulomb}^2}$.
Q_1, Q_2 = the charges of the particles in coulombs.
d = the separation in meters.

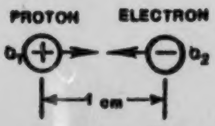
The use of Equation 1 in solving problems is illustrated in Examples A and B:

EXAMPLE A : ATTRACTIVE FORCE BETWEEN UNLIKE CHARGES

Given: An electron is seperated from a proton by a distance of one cm.

Find: The force with which the electron pulls on the proton and with which the proton pulls on the electron.

Solution: The electron pulls on the proton with the same force that the proton pulls on the electron. This force is given by Equation 1. Thus,



$$F = \frac{kQ_1Q_2}{d^2}, \qquad k = 9 \times 10^9 \, \frac{\text{N}\cdot\text{m}}{\text{C}^2}$$

$Q_1 = Q_{\text{proton}} = 1.602 \times 10^{-19}\text{C}$
(See Table 1.)
$Q_2 = Q_{\text{electron}} = 1.602 \times 10^{-19}\text{C}$
(See Table 1.)
$d = 1 \text{ cm} = 10^{-2} \text{ m}$

$$\therefore F = 9 \times 10^9 \, \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}} \times \frac{1.602 \times 10^{-19}\cancel{\text{C}} \times 1.602 \times 10^{-19}\cancel{\text{C}}}{(10^{-2})^2\cancel{\text{m}^2}}$$

$$\therefore F = 2.3 \times 10^{-24} \text{ N}$$

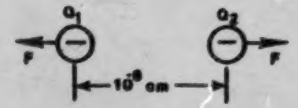
Since about 4.5 N are required to lift a one-pound weight, the electrical force here is very small. Electrical forces between charged particles are often very small.

EXAMPLE B: REPULSIVE FORCES BETWEEN LIKE CHARGES

Given: Two electrons in adjoining copper atoms are separted by 10^{-8} cm.

Find: The force with which the electrons repel each other.

Solution:



(From Equation 1.)

$$\therefore F = \frac{kQ_1Q_2}{d^2}$$

$k = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$

$Q_1 = Q_2 = 1.602 \times 10^{-19}$ C (See Table 1.)

$d = 10^{-8}$ cm $= 10^{-10}$ m

$$\therefore F = \frac{(9 \times 10^9 \frac{N \cdot m^2}{\cancel{C^2}})(1.602 \times 10^{-19})^2 \cancel{C^2}}{(10^{-10})^2 \cancel{m^2}} =$$

$$F = 2.3 \times 10^{-8} \text{ N}$$

This force is ten million billion times larger than the force in Example A, but is still small compared to one newton.

Notice that although the forces are often very small, as shown in both Examples A and B, they often act on very small particles (see masses of electron and proton in Table 1, for example) and therefore are quite effective in pushing or pulling these particles around.

ELECTRIC FIELDS

An electric charge in space presumably will change the space around it; the charge sets up an ELECTRIC FIELD. The electric field intensity E at a position P is defined as equal to the electric force per unit positive charge at position P, as shown by Equation 2.

$$\vec{E} = \frac{\vec{F}}{Q} \tag{2}$$

where
$\vec{E}$ = the electric field intensity in N/C at position P.
$\vec{F}$ = the force in newtons on a charge Q at position P.
Q = the charge at position P.

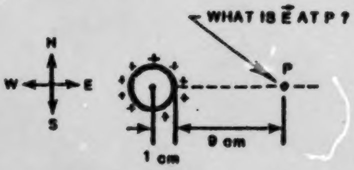
The electric field intensity $\vec{E}$ and force $\vec{F}$ are written with an arrow to show that each is a vector. Since a vector has both a magnitude and a direction, the electric field intensity $\vec{E}$ will have both a magnitude and direction, just as does the force $\vec{F}$. The magnitude of $\vec{E}$ (in newtons/coulomb) always will be given by the ratio F/Q in Equation 2. Its direction always will be given by the direction of the electric force $\vec{F}$ on a small positive test charge Q placed at the position in question. The use of Equation 2 is illssustrated in Example C.

EXAMPLE C : ELECTRIC FIELD NEAR A CHARGED SPHERE

Given: A hollow metal sphere is charged to a high voltage, thereby producing an electric field in the space around it. The sphere has a charge of +10 coulombs and a radius of one centimeter.

Find: The electric field intensity $\vec{E}$ at a position 9 cm due east of the sphere.

Solution:



Note: Any charged conducting sphere can be replaced by a single-point charge of the same maghitude at the center of the sphere, so long as force and field calculations external to the sphere are to be determined; thus, point P is effectively 10 cm away from a 10 coulomb point charge. Now arbitrarily choose 0.01 coulombs as the small test charge at P. The choice of 0.01 coulombs for the test charge is not important, since its value does not affect the final value of E calculated below. (See note below.)

Then from Equation 1:

$$\text{(a)}\quad F = \frac{k\, Q_{sph} Q_{test}}{d^2}$$

$$k = 9 \times 10^9\ \frac{N \cdot m^2}{C^2}$$

$$Q_{sph} = 10 \text{ coul}$$

$$Q_{test} = 0.01 \text{ coul}$$

$$d^1 = 1 \text{ cm} + 9 \text{ cm} = 10 \text{ cm}$$

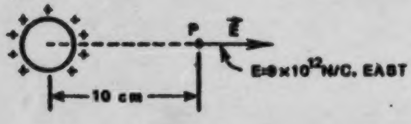
$$d^2 = 0.01 \text{ m}^2$$

$$F = \frac{(9 \times 10^9)\ (10)\ \left(\frac{\cancel{1}}{\cancel{100}}\right)}{\frac{\cancel{1}}{\cancel{100}}} = 9 \times 10^{10} \text{ N}$$

and from Equation 2:

(b) $E = \frac{F}{Q_{test}} = \frac{9 \times 10^{10}}{1/100\ C}\ N = 9 \times 10^{12}\ N/C$

The complete answer for the required electric field intensity $\vec{E}$ at point P would then be 9×10^{12} N/C in an eastward direction. The direction of the field at a point is always taken as the direction of the force on a small positive test charge placed at that point. Thus:



Notice that the choice of 0.01 coulombs for the magnitude of the charge does not affect the final answer. The effect of Q_{test} on the answer is part (a) is cancelled out in part (b); therefore, a value of Q_{test} = 1 C can be taken more easily.

As illustrated in Example C, the direction of the electric field at any point is the direction of the force on a positive charge at that point. The advantage of having the concept of an E field is that one can measure the strength of the electric force without knowing anything about the charges that cause the force. Since forces which arise from many charges can be mathematically quite complicated, to work with the electric field is usually simpler than with the detailed charge configurations and the resulting forces between them.

In portraying an electric field around a charge distribution, whether the distribution is a single point charge or an assembly of charges on a conductor, imaginary lines are used to represent the electric field, both in direction and strength. These lines always begin on positive charges and end on negative charges. The direction of a line to any position in space around the charges is the direction of the force on a positive charge at that position. The strength of the field is represented by the density of lines; that is, the closer the lines, the stronger the field, as shown by the four separate drawings in Figure 2. Different representations of an electric field near two positions, P_1 and P_2, are shown in the figure.

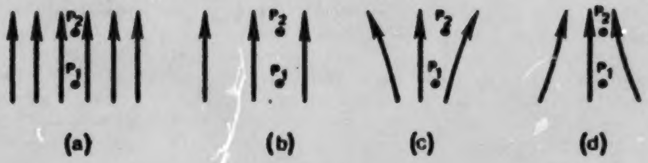
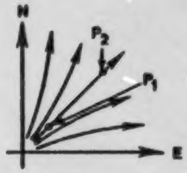


FIG. 2

Figure 2a and 2b represent uniform or constant-strength fields, since the field lines remain the same distance apart. Figure 2a represents a field twice as strong as Figure 2b, since the lines are spaced twice as close together. In each case, Figure 2a and 2b, the field is constant, and the force on a positive charge at points P_1 and P_2 would be identical and directed upwards. In Figure 2c the field weakens in going from P_1 to P_2, since the lines spread. In Figure 2d the field intensifies in going from P_1 to P_2 since the lines crowd closer together. In Figure 2c the direction of the field at P_2 is mostly north but somewhat east. In Figure 2d the field is mostly north but slightly west. These principles are used to solve a problem in Example D.

EXAMPLE D : ELECTRIC FIELD PROBLEM

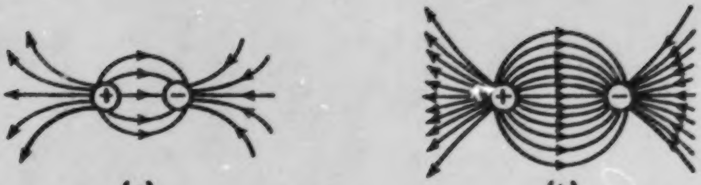
Given: Electric field configuration as shown. A point charge of $+2 \times 10^{-7}$ coulombs at position P_2 experiences a force of 6 newtons.



Find:
(a) The electric field at P_2.
(b) Whether the same charge is greater of smaller at positon P_1.

Solution:
(a) $E = \frac{F}{Q} = \frac{6\ N}{2 \times 10^{-7} C} = 3 \times 10^{7}$ N/C, direction northeast.
(b) The same charge is greater at P_1, since field E is greater (lines are closer together).

Electric field lines around various charge configurations are shown in Figure 3. Electric field lines at any position always point in the direction of force on a small positive charge placed at that point. Figure 3a shows the field lines around an equal positive and negative charge-pair. In Figure 3b field lines are shown around another equal positive and negative charge pair as in a; but here the charges are larger, leading to a stronger electric field (thus lines are drawn closer together). Figure 3c shows the cross section of two oppositely-charged parallel plates. There are fringing, or bulging, effects of the field at the plate edges, and the field lines are evenly spaced at the center and throughout the interior region between the plates. These features indicate

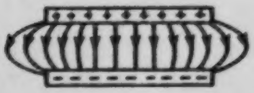


(a) (b)

a constant strength field, as in Figure 2a and 2b.

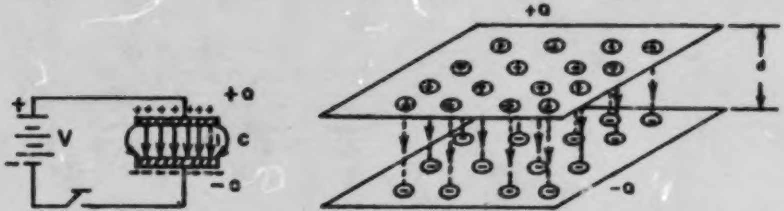
PARALLEL PLATE CAPACITORS

A capacitor is defined as two conducting surfaces separated by a dielectric (insulating) medium. The simplest version of a capacitor, and one which illustrates the important relationships, is the so-called parallel plate air-capacitor. Figure 4a shows the charging circuit, and Figure 4b shows a three-dimensional view of the capacitor alone.



(c)

FIG. 3



(a) CHARGING CIRCUIT

(b) PARALLEL PLATE CAPACITOR WITH FIELD LINES

FIG. 4

In Figure 4a, a battery with voltage V is used to charge a capacitor to a potential difference of V volts with a positive charge of +Q coulombs on one plate and a negative charge of -Q coulombs on the other plate. If the capacitor has a capacitance, the relationship between Q, C and V is given by Equation 3.

$$Q = CV \qquad (3)$$

where

C = capacitance in farads.

V = volts.

In Figure 4c, the constant E field (fringing near the edges is neglected here) between the plates is shown by the dotted line with the direction pointing from +Q (top plate) to -Q (bottom plate). If the plates were charged with a battery of voltage V and the distance, in meters, between the plates is d, then the strength of the E field is given by Equation 4.

$$E = \frac{V}{d} \tag{4}$$

where

E = electric field strength, expressed equally well in volts/meter or newtons/coulomb.

Example E illustrates the application of Equations 3 and 4 in a typical capacitor problem.

EXAMPLE E: CAPACITOR PROBLEM

Given: A parallel-plate capacitor whose plates are separated by 1 cm is charged to a voltage of 50 volts.

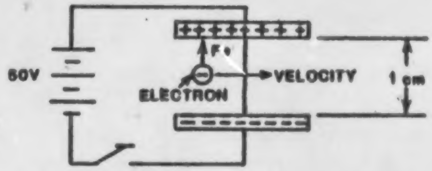
Find:

(a) The electric field strength between the plates.

(b) The force F_e exerted on an electron by the electric field if the electron is passing through the air space between the capacitor?

(c) If the plates each carry a charge of 50×10^{-6} C, what is the capacitance of the capacitor?

Solution:



(a) From Equation 4, $E = \frac{V}{d}$:

$$E = \frac{50V}{10^{-2}m} = 5000\ V/m = 5000\ N/C$$

(b) From Equation 2, $E = \frac{F}{Q}$ $\quad Q = 1.6 \times 10^{-19}$ C (See Table 1.)

$F = 5000$ N/C.

$$F_e = EQ = (5000\ N/C)(1.6 \times 10^{-19}\ C)$$

$F_e = 8 \times 10^{-16}$ N, directed upward toward + plate.

(c) From Equation 3, $Q = CV$

$$C = \frac{Q}{V} = \frac{50 \times 10^{-6}\ C}{50V} = 10^{-6}\ \frac{C}{V} = 10^{-6}\ \text{farad}$$

C = 1 microfarad

APPLICATIONS OF ELECTRIC FIELDS AND CAPACITORS

Example E indicates that a charged particle, such as an electron, experiences a force because of the existence of an electric field in the region between the parallel plates of the capacitor. Such a force can be used to deflect moving electrons and thereby change their paths. This basic idea is used in controlling the path of electrons from the electron gun to the screen in a television tube. Figure 5 illustrates deflection of an electron beam with charged capacitor plates in a television tube/cathode ray tube.

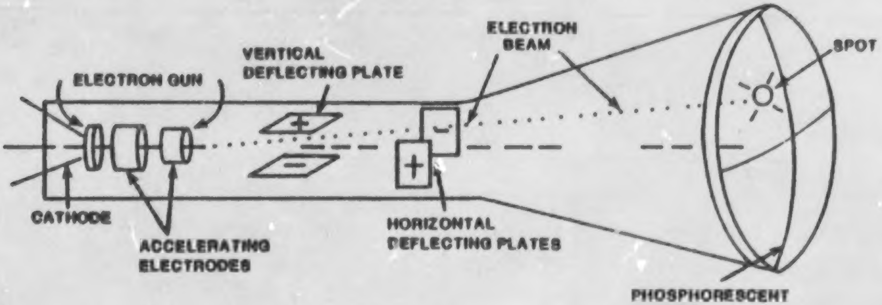


FIG. 5

The electron beam leaves the accelerating electrodes along the central axis of the tube. If the vertical deflecting plates are charged in such a manner that the top plate is positive and the bottom plate is negative, at the instant the electron passes through, it will be deflected upwards. Then, if the horizontal deflecting plates are charged with front plate positive, rear plate negative, as the electron passes between these plates, it will be deflected further toward the positive plate. The combined deflections will result in the electron beam striking the screen in the upper left quadrant, as shown. By changing voltage on the horizontal and vertical deflecting plates in a pre-arranged manner, the electron beam can be directed to various positions on the screen. In each instance, as in Example E, the existence of an electric field between the capacitor plates gives rise to an electric force on the charged particles (electron beam), which in turn deflects the motion of the particles.

In another basic application, known as electrostatic precipitation, the strong electric field between oppositely charged capacitor plates is used to precipitate out smoke, dust, fume, fog, haze, smog, aerosols, etc., from polluted air. In these days of environmental control, the purification of air in industrial environments is of great importance.

In the electrical precipitation process, three fundamental steps are required:

(a) electrical charging of foreign particles, that is, pollution.
(b) collection or deposition of charged particles in an electric field.
(c) removal of precipitated material from collecting electrodes to an external receptacle.

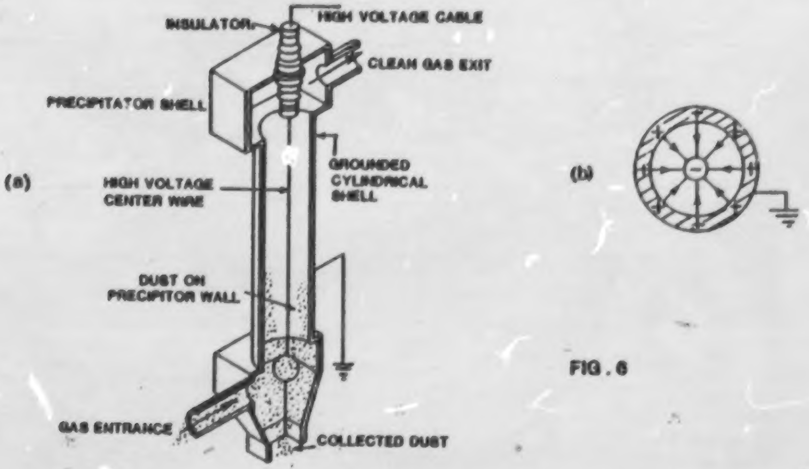

INSULATOR
HIGH VOLTAGE CABLE
CLEAN GAS EXIT
PRECIPITATOR SHELL
(a)
HIGH VOLTAGE CENTER WIRE
GROUNDED CYLINDRICAL SHELL
DUST ON PRECIPITOR WALL
GAS ENTRANCE
COLLECTED DUST
(b)

FIG. 6

Figure 6a illustrates the general configuration in a single-state precipitator. In this arrangement the capacitor differs in geometry from the parallel-plate capacitors described above. The capacitor here consists of a high-voltage wire located along the central axis of a long pipe (cylinder). The center wire forms one "plate" of the capacitor; the cylinder wall forms the other "plate." Figure 6b shows the cylindrical capacitor in cross section, with the appropriate electric field lines.

The high voltage on the central wire (usually from 10,000 to 100,000 volts) creates a corona discharge region near the wire. The particle pollutants passing through the corona region pick up a negative charge of 300 electron charges for one-micron size particles and as high as 30,000 electron charges for 10-micron size particles. The strong electric field between the central wire and the cylindrical wall exerts a considerable force on the charged particles, driving them from the region of the corona discharge towards and onto the cylindrical walls. As shown in Figure 6, the pollutants are collected on the walls. The foreign particles are then shaken loose by some clever mechanical means and collected at the bottom of the precipitator. Clean air (or gas) is drawn out through the top.

A somewhat more sophisticated, two-stage arrangement of an electrostatic precipitator is shown in Figure 7. The left stage is the charging region where the pollutants flowing by the corona wires B and grounded cylinders G pick up electric charge. The right stage contains a series of parallel-plate capacitors labeled alternately C and D. The grounded collector plates are labeled C, the charged plates D. As the charged pollutants pass through the electric field region between plates C and D, the particles are deflected, collected on the plate surfaces C and removed. Purified air then emerges further downstream.

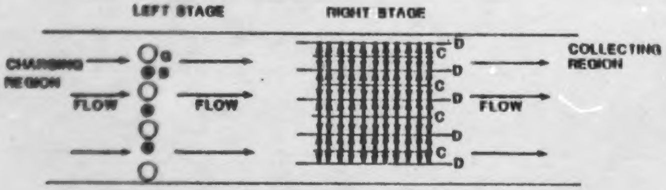


FIG. 7

SUMMARY

Electric forces between electrically charged particles form the basis of operation for various electronic components and devices. Such electric force produces electric fields around a component such as a capacitor. Some electric fields are used for deflecting electron beams in cathode-ray tubes; others, for electrostatic precipitation and other purposes.

LINE PATTERNS OF ELECTRIC FIELDS

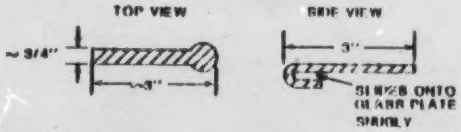
MATERIALS

8 x 8-inch glass plate.
4 rubber suction cups (or 4 rubber stoppers).
1 induction coil or Wimshurst-type electrostatic machine.
Assorted electrodes made of copper or aluminum -
(see experiment for details).
Chopped, clean straight hair (1/4 to 1/2 inch lengths).

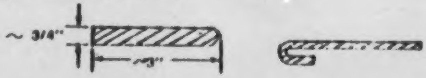
PROCEDURE

This experiment is designed to show the electric field line patterns around various charged electrodes. The electrodes to be used should be constructed from thin aluminum or copper metal stock; however, if these are unavailable, aluminum foil can be used. The shapes required, with essential dimensions, are shown below in Figure 8:

(a) Circular-end electrodes (2 each)



(b) Pointed-end electrodes (2 each)



(c) Parallel cross-strip electrodes (2 each) to simulate parallel-plate capacitors

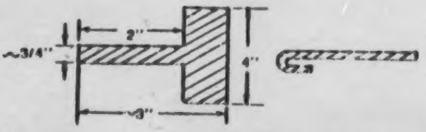


FIG. 8

1. Arrange apparatus as shown in Figure 9 with a clean glass plate lying on four clean rubber stoppers or four clean rubber suction cups. (Cleanliness is required to prevent leakage of charge.)

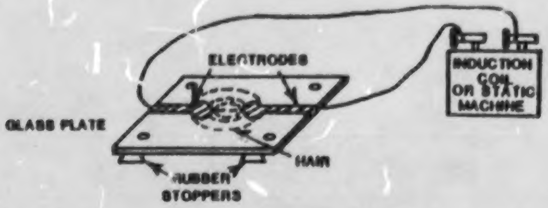


FIG. 9

2. Center the circular end electrodes on the glass plate as shown, ensuring that end clips fit snugly onto glass plate.

3. Attach wires from terminals of induction coil or static machine to electrodes.

4. Plug in induction coil or crank static machine to ensure a high voltage on electrodes. (Caution: Avoid touching electrodes and lead wires in operation.)

5. While voltage is on and electrodes are charged, sprinkle chopped hair lightly in central region between electrodes. Chopped hair should align itself along clearly-visible field lines. Tap glass plate lightly and repeatedly with rubber end of a pencil to facilitate orientation of chopped hairs.

6. Draw a top view of glass plate, electrodes and field lines (roughly to scale) on a clean sheet of paper. The drawing should be roughly to scale and should indicate clearly the electric field line pattern.

7. Remove induction coil or static machine wires, remove electrodes and collect chopped hair. Repeat steps 2 through 6 with the pointed-end electrodes and parallel cross-strip electrodes.

8. Explain any differences between the field patterns of step 6 and step 7.

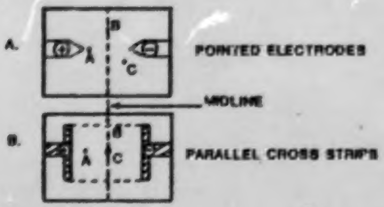
SELF CHECK

1. State one similarity and one difference in the charge of an electron and that of a proton.

2. Determine the force in newtons experienced by two protons separated by a distance of one angstrom (1A° = 10^{-10} m).

3. A parallel-plate capacitor of 10μ farad capacitance is charged to a voltage of 100 volts. The plates are separated by one millimeter of air. What is the electric field in the region between the plates in volts/meters? In newtons/coulomb? How much charge in coulombs was deposited on each plate in the charging process? If an alpha particle (see Table I) is moving along a midplane between the capacitor plates, what is the force on the alpha particle? Is the force the same at the ends of the capacitor as it is near the center?

4. Refer to the electric fields mapped in the laboratory section of this module. For the electrode geometries shown in the adjoining figure and for the point A, B, and C, as marked, indicate with arrows the relative strength of the field, the direction of the electric field and the direction of the force on an electron placed at those points. Assume electrodes charged as shown.



5. Define:

   Coulomb force.
   Electric field line.
   Electric field.
   Parallel plate capacitor.

6. In words, express the basic equation for the following:

   Force between two charged particles.
   Relationship between electric field $\vec{E}$, charge Q, force $\vec{F}$.
   Relationship between E, V and d for a parallel plate capacitor.
   Relationship between Q, C and V for a parallel plate capacitor.

7. Describe how parallel plate capacitors and electric fields play central roles in television tubes and electrostatic precipitators.

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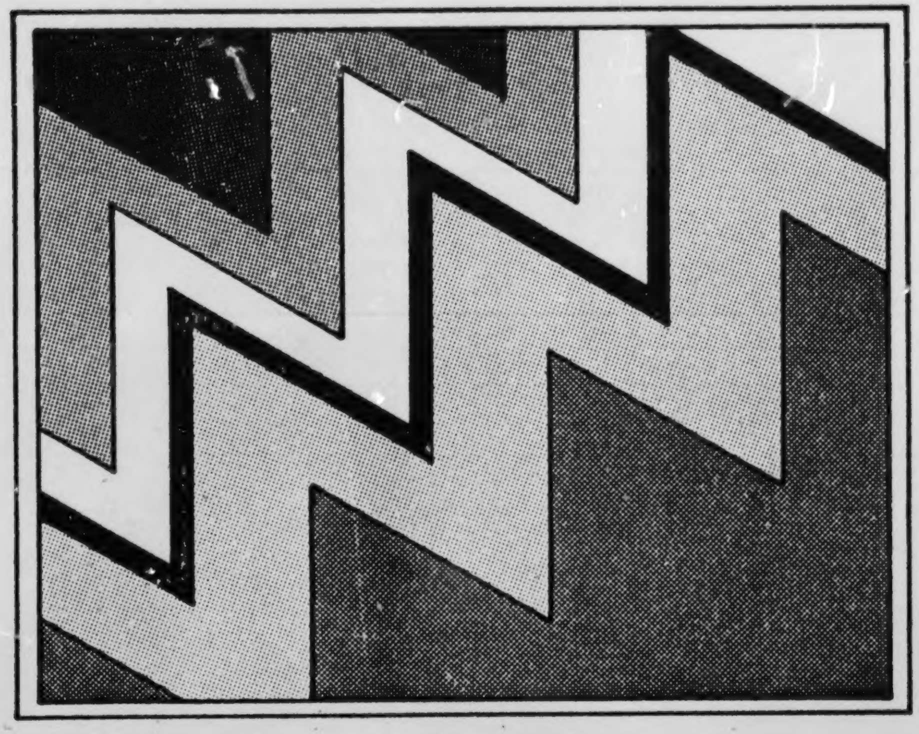
UNIFIED TECHNICAL CONCEPTS

MODULE 1 E 4

MAGNETIC FORCES AND FIELDS



ELECTRICAL APPLICATION OF FORCE

Technical Education Research Center – Southwest

TERC - SW

INTRODUCTION

Magnetic forces and fields are vital to the operation of generators, motors, induction coils, inductors, particle accelerators and to the general field of electronics. This module discusses the origin of magnetic forces, the representation of magnetic forces in terms of magnetic fields, and the mapping of magnetic fields around bar magnets, horseshoe magnets, iron-core solenoids and long, straight wires. It also discusses how forces develop on current-carrying conductors in the presence of a magnetic field and how this principle is used in the construction of generators and motors. In the laboratory, the student will map magnetic fields, observe the results of forces upon current-carrying conductors in magnetic fields, and will construct and operate a simple electromagnet.

PREREQUISITES

Student should have completed Concept Module I-0, "Force," and Application Module IE3, "Electric Forces and Fields."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Explain how moving charges produce magnetic fields in a permanent magnet and electromagnet.
- Explain the nature of attractive and repulsive forces between north and south poles of permanent magnets.
- Draw the magnetic-field lines around bar magnets, horseshoe magnets, a long, straight wire carrying a current and a solenoid carrying a current.
- Predict the location of the "equivalent" north and south poles of a solenoid from the direction of flow of electron current through the windings.
- Given the direction of the magnetic field, the relative field

strength and direction of motion of a charged particle in that field, predict the direction and relative strength of magnetic force exerted on the particle by the magnetic field.

- Given the direction of a magnetic field and the direction of motion of a conducting wire through that field, predict direction of electron current flow in the wire.

- Given the direction of a magnetic field and direction of current flow along a wire in that field, predict direction of the magnetic force on the wire.

- Explain with use of diagrams how the combination of a magnetic field and a moving, current-carrying conductor leads to the construction of a motor and a generator.

- Given the appropriate equipment, (1) show the magnetic-field patterns around different geometries of permanent magnets; (2) demonstrate the interaction between current-carrying conductors and magnetic fields involving long, parallel wires carrying currents; and (3) construct a simple lifting electromagnet, and determine its lifting force.

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THE ORIGIN OF MAGNETISM

First experiences with magnetism and magnetic forces usually come from contact with bar magnets and compasses. Bar magnets and compasses have north and south poles that strongly attract one another, while two north poles or two south poles strongly repel one another. Also, some magnets, such as lodestone, are "permanent" in behaviour; that is, they exhibit magnetic properties seemingly forever. On the other hand, some magnets are "temporary" in behaviour, exhibiting magnetic properties only while a current is flowing near a "magnetic substance" such as soft iron. Electromagnets - wires wrapped around soft iron cores - are good examples of temporary magnets.

The origin of natural, permanent magnetism - as found in the substance called lodestone - or of temporary magnetism, as produced in electromagnets, can be traced in either case to the existence of microscopic atomic currents deep within matter itself. A tiny current, such as that formed by an electron whirling around inside an atom, gives rise to a magnetic force field in a region surrounding the electron's orbit.

In Figure 1a, an electron is whirling around in an orbit with axis tilted as shown. In Figure 1b, the magnetic field created by the movement of the electron in its orbit is shown as if it were produced by a tiny magnet aligned along the same axis with appropriate N and S poles.

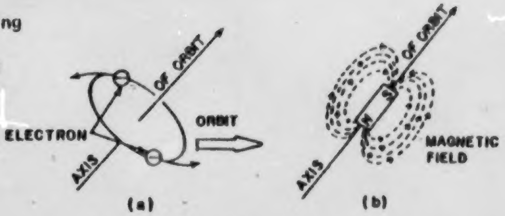
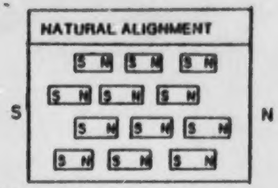
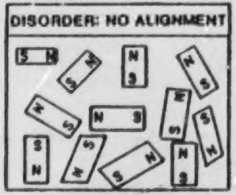


FIG. 1

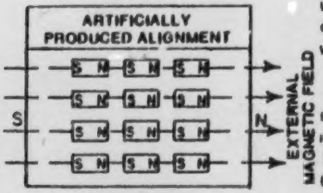
In effect, as illustrated in Figure 1, the tiny electron current can be thought of as a tiny bar magnet with an appropriate N and S pole giving rise to a magnetic field around the bar magnet. In natural or permanent magnets such as lodestone, therefore, all must be permanently aligned in such a manner that all the tiny equivalent magnets will add up to one large effecttive magnet. For permanent magnets, then, the key idea is clearly in the natural alignment of clusters of atoms - called magnetic domains - which in turn assures that the overall addition of billions and billions of the tiny equivalent magnets will result in one large effective magnet. (See Figure 2a.) On the other hand, initially unmagnetized materials somehow must lack the precise alignment present in the magnetic domains of permanent magnets; that is, here the alignment must be random in nature. (See Figure 2b.) The addition of billions and billions of randomly-ordered, unaligned, tiny equivalent magnets results in an overall effect of no magnetism, as with ordinary soft iron. But in unmagnetized substances - such as soft iron - which can be magnetized, these billions and billions of initially-disordered tiny magnets can be



(a) PERMANENT MAGNET (STRONG MAGNETIC FIELD)



(b) UNMAGNETIZED SUBSTANCE (NO MAGNETIC FIELD)



(c) MAGNETIZED SUBSTANCE (STRONG MAGNETIC FIELD)

FIG. 2

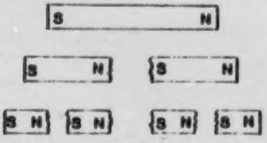


FIG. 4

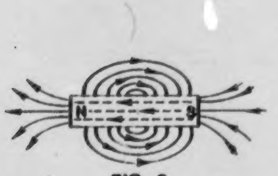


FIG. 3

rearranged and aligned (as in permanent magnets) by an external magnetic force threading through the substance. In this manner, an unmagnetized substance often can be magnetized as shown in Figure 2c.

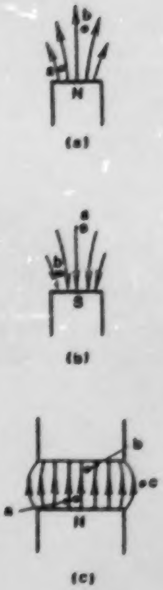
MAGNETIC FORCE AND FIELD

As indicated in the introductory discussion of the origin of magnetism, all magnetic substances can be represented in terms of two poles, a north (N) and south (S) pole. Any attempt continually to subdivide a bar magnet - as shown in Figure 3 - with the intention of separating and isolating one pole from the other, is doomed to failure. Subdivision of a magnet simply results in a smaller magnet of less strength, but always with two poles, N and S, just as in the original magnet. To this date, although scientists have been trying for years, no one has succeeded in isolating a free magnetic pole, either a south or north pole. Thus one is forced to work with both N and S poles when studying magnetic forces. (When electric forces were studied in Module IE3, no trouble separating electric charge of positive and negative character from one another was encountered.)

If two simple bar magnets are arranged with their north poles (or south poles) next to one another, the magnets will strongly repel each other. This observation is summarized by the statement, "like magnetic poles repel." On the other hand, if the same two bar magnets are arranged with their N and S poles next to each other, they will attract each other strongly, which demonstrates that "unlike magnetic poles attract."

Figure 4 shows a single bar magnet with clearly labeled N and S poles. Drawn in the space surrounding the magnet are a series of lines to represent the direction of force on a tiny N pole of a very long magnet. (The force on the distant south pole is unimportant here.) The series of lines, just as in the discussion of electric charges and electric force in Module IE3, constitutes a field, in this case, a magnetic field. Figure 4 shows the magneltc-field lines around a simple bar magnet. Solid lines represent the field outside the magnet; dotted lines represent the field inside the magnet. The lines always point in the direction of the force on a small north pole located at the given point in space. At the left end (N) of the bar magnet in Figure 4, field lines are directed away from the N pole (since two N poles repel); and at the right end (S), the field lines are directed toward the S pole (since a N and S pole attract). And here, as in Module IE3 on the electric field, the strength of the magnetic field is indicated by the density of lines drawn. Closely-spaced lines indicate a strong field; widely-spaced lines indicate a weak field. Uniformly-spaced lines indicate a magnetic field of constant strength, and diverging or converging field lines indicate a decreasing or increasing magnetic field strength, respectively.

In Figure 5a, the magnetic field weakens in going from point a to b since the field lines spread. In Figure 5b, the field intensifies in going from point a to point b, since the lines are more densely packed. In Figure 5c, the field remains constant in going from a to b, since the lines there remain equally spaced. In going from a to c in Figure 5c, the field would, of course, weaken.



(a)

(b)

(c)

FIG. 5

The pattern of magnetic-field lines for various geometries is shown in Figure 6. The first two drawings, 6a and 6b, illustrate the magnetic field lines near adjoining N-S poles and N-N poles. The third drawing maps the field lines near a horseshoe magnet; that is, a simple bar magnet bent in a U-shape. The fourth drawing shows the field lines around a long, straight wire carrying electron current, and the fifth sketch shows the field lines around an air core and iron core solenoid (coil).

The patterns of the field lines in Figures 6a and 6b are vastly different, and the patterns in 6a and 6c are quite similar. Note that the magnetic field created by a long wire carrying a current I is a series of concentric rings encircling the wire, as shown in Figure 6d. The direction of the magnetic field lines can be determined easily as follows. If the thumb of the left hand is placed along the wire in the direction of electron current flow, the fingers of the left hand wrap themselves around the wire in the direction of the circular magnetic field lines. In Figure 6d, therefore, the thumb would point downwards in the direction of current flow, and the fingers would encircle the wire in a counterclockwise direction (looking down from the top), as shown.

For the ion-core and air-core solenoids (Figure 6e), the shape of magnetic field lines is similar to that shown in Figure 4 for a simple bar magnet. For electron current flowing around the core of the solenoid as shown, each solenoid "behaves" like a simple bar magnet, with a N pole at the left end and a S pole at the right end. Which end of the solenoid behaves like a N pole can be determined from the following simple rule. If the fingers of the left hand are wrapped around the solenoid in the direction of electron current flow, the extended thumb will point to the end of the solenoid which behaves like a N pole. If this rule is applied to Figure 6e, the left ends of the solenoids are the indicated N poles.

A comparison of magnetic-field sketches for the air-core and iron-core solenoids shows that the field is intensified in the center of the iron-core solenoid by the addition of the iron core. A stronger magnetic field can be obtained with a solenoid of given current and windings simply by inserting an iron core through the solenoid.

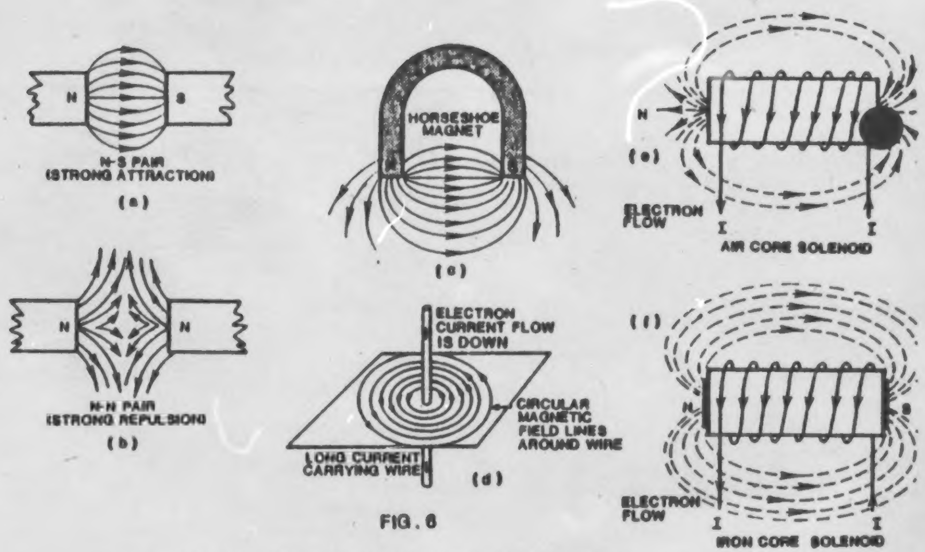

N
S
N-S PAIR (STRONG ATTRACTION)
(a)
N
N
N-N PAIR (STRONG REPULSION)
(b)
HORSESHOE MAGNET
(c)
ELECTRON CURRENT FLOW IS DOWN
CIRCULAR MAGNETIC FIELD LINES AROUND WIRE
LONG CURRENT CARRYING WIRE
(d)
N
(e)
ELECTRON FLOW
I
I
AIR CORE SOLENOID
(f)
N
S
ELECTRON FLOW
I
I
IRON CORE SOLENOID

FIG. 6

MAGNETIC FORCES ON MOVING CHARGES

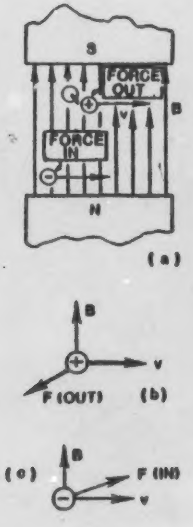

S
FORCE OUT
v
B
FORCE IN
N
(a)
B
v
F (OUT)
(b)
(c)
B
F (IN)
v

FIG. 7

Simple forces exerted on electric charges moving in a magnetic field are essential for the operation of the generator and the motor.

In the upper half of Figure 7a, a positively-charged particle of magnitude +Q is shown moving through a uniform magnetic field of strength B with a velocity v. The magnetic field is directed upwards. In this configuration, the force F exerted on the positive charge is directed OUT of the page toward the reader (Figure 7b). If the particle was negatively charged, everything else being the same, the force on the negative charge would be directed IN to the page, away from the reader. This situation is pictured in Figure 7c and the lower half of Figure 7a. The magnetic force F for a charged particle Q moving with velocity v in a magnetic field B is always directed at right angles to both the direction of the field $\vec{B}$ and the velocity $\vec{v}$. The direction of force on a moving POSITIVE charge can be determined from the following simple rule:

(i) First, locate the plane determined by vectors $\vec{v}$ and $\vec{B}$ and arrange $\vec{v}$ and $\vec{B}$ with their tails at the same position P. Suppose $\vec{v}$ is up and $\vec{B}$ is to the right, as in Figure 8a.

(ii) Next, imagine a "right-handed" screwdriver oriented perpendicularly to the plane of $\vec{v}$ and $\vec{B}$, with its blade positioned at P. Now imagine turning the screwdriver in the same direction which is required to rotate vector $\vec{v}$ into $\vec{B}$ (with B held fixed) through the smaller angle separating $\vec{v}$ and $\vec{B}$. (See Figure 8b.)

(iii) The direction of advance (or recession) of a screw being driven by the rotating screwdriver gives the direction of force on the positive particle. In Figure 8b, the clockwise rotation of the screwdriver would cause the screw to advance; therefore, the direction of force is into the page.

(iv) If the particle had been negative, the rule would have been applied as explained in the three steps above, except that the direction obtained in (iii) simply would have been reversed; that is, if step (iii) had indicated a force IN for a positive particle, the direction would have been OUT for a negative particle.

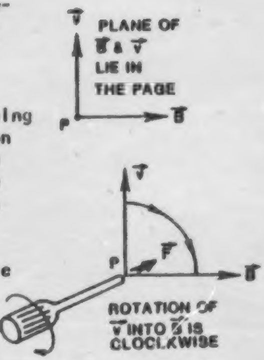

$\vec{v}$
PLANE OF $\vec{B}$ & $\vec{v}$ LIE IN THE PAGE
P
$\vec{B}$
$\vec{v}$
P
$\vec{F}$
$\vec{B}$
ROTATION OF $\vec{v}$ INTO $\vec{B}$ IS CLOCKWISE

FIG. 8

The amount of force on the charged particle depends upon the velocity component perpendicular to the magnetic field. There is no force on a particle moving parallel to the magnetic field.

GENERATORS AND MOTORS

The force on a moving charge in a magnetic field lies at the heart of the principle of operation of the electric generator and electric motor. In the generator, external mechanical forces are used to push a conducting wire through a strong magnetic field, thereby inducing magnetic forces on the charge parallel to the wire and generating a current in the wire. In the motor, electric current (moving charge) is forced through a conducting wire located in a strong magnetic field. The magnetic forces in this case act on the charges in the wire, but in a direction perpendicular to the wire, thereby pushing the wire. In generators, external mechanical power is converted into electric power. In motors, external electric power is converted to mechanical power.

The basic idea behind the generator is illustrated in Figure 9.

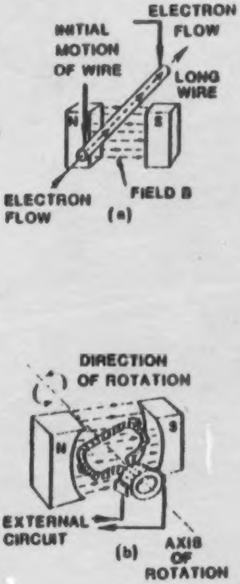


FIG. 9

In Figure 9a, a long, straight conducting wire filled with electrons and initially carrying no current, is pushed downward into a magnetic field B directed from left to right. When the wire is forced downward, each electron in the wire also moves downward with the wire: A negative charge moves downward in a magnetic field directed from left to right, representing electric charge moving perpendicularly in a magnetic field. In accordance with Figure 7, the moving charge (electron) must experience a magnetic force. Application of the screwdriver rule explained in connection with Figure 8 indicates that magnetic force upon each electron must be directed along the wire INTO the page, away from the reader. Motion of the wire in the magnetic field therefore generates an electron current flow in the wire.

If the wire shown in Figure 9a is part of a loop shown in Figure 9b and if the loop rotates as shown, electron current can be generated in the left and right portions of the loop. Current is generated in opposite directions as shown, to provide continuous current flow to an external circuit. This design, then, is the simplest example of a working generator.

The electric motor can be thought of as a generator working in reverse. In Figure 10, the wire is initially stationary, situated in the magnetic field as shown. Now a current (electron) is forced through the wire, out toward the reader. This process again constitutes electric charge moving in a magnetic field; in accordance with Figures 7 and 8, a magnetic force in a specified direction will act upon the moving charge. Application of the right-handed screwdriver rule shows that the force on the moving charge is directed downwards. Since this moving charge is perpendicular to the wire, the wire experiences a force and moves downward. Thus electrical power used initially to force current through the conductor therefore is transformed into the mechanical power in the moving wire. If the wire in Figure 10 is made part of a loop and fastened to a rotating shaft, somewhat as shown in Figure 9b, the mechanical power generated in the rotating loop can be used to do useful external work.

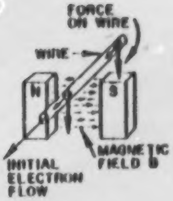


FIG. 10

SUMMARY

Magnetic forces and fields play an important role in electricity and in electronics. Electric motors, generators, coils, inductors, etc., depend upon such forces and fields. In the case of motors and generators, magnetic fields exert forces upon electric charges moving through them.

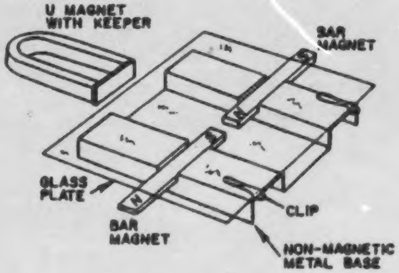
LABORATORY

MAPPING MAGNETIC FIELDS

MATERIALS

2 bar magnets.
1 U magnet.
Non-magnetic metal base (see Figure 11).
Double-strength window glass plate, 8 x 11 inches.
Iron filings in sifter-type jar.

PROCEDURE



1. Refer to Figure 11. Slip glass plate under the metal clips in such a manner that it rests securely on non-magnetic metal base.

2. Slide bar magnets into slots shown with S pole next to N pole, as shown. Sprinkle iron filings onto glass plate near center, over the N-S pole pair. Tap glass plate several times until the magnetic field lines stand out. Compare with Figure 6a. Remove glass plate; carefully brush iron filings from glass plate; and replace glass plate.

3. Reverse orinentation of one of the bar magnets in (2) above so that a N-N or S-S pole pair exists under the center of glass plate. Sprinkle iron filings on glass plate and tap as necessary. Compare magnetic-field line pattern exhibited by iron filings with Figure 6b. Again, remove glass plate; brush off filings; and replace glass plate.

4. Remove bar magnets from side slots, and slide U magnet (without keeper) into large center slot with two poles underneath center section of plate. Sprinkle iron filings on plate and tap plate as needed. Examine magnetic field lines and compare with Figure 6c.

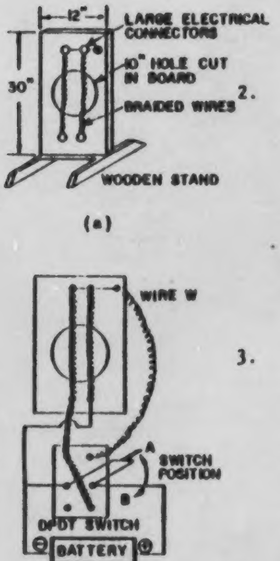
5. Why do the line patterns in steps 3, 4 and 5 differ?

FORCES BETWEEN CURRENTS END FIELDS

MATERIALS

1 wooden stand, 12 x 30 inches. (See Figure 12.)
2 flexible braid wires, 25 inches long x 1/16 inch diameter (use braided ground sheath in a coaxial cable if braid wires are not readily available).
1 double-pole, double-throw switch.
1 12-volt lead-acid auto battery.
Heavy-duty copper wire and battery clips.

PROCEDURE



(a)

(b)

FIG. 12

1. Refer to Figure 12. In addition to two vertical braided wires stretched over 10-inch hole, as shown in Figure 12, all other connecting wires to the DPDT switch and 12-volt auto battery must be heavy gauge, capable of carrying large currents.

2. Arrange the apparatus - all home built - as shown in Figure 12a, wired as shown in Figure 12b. Caustion: Leave DPDT switch in open position. (Wires from battery to DPDT switch should be heavy battery cables with lugs on one end and spring clips on the other.) Wire W in Figure 12b should be connected from top of board to the DPDT switch in such a manner as not to be either parallel to or too near two vertical braided wires. The two braided wires, each about 25 inches long, should not be stretched too tightly between upper and lower connections on wooden board; otherwise, sideways attraction and repulsion will not be visible.

3. If electron current moves through each wire in same direction (say, up - this is position A of the DPDT switch) the two wires will be attracted and move towards each other. If electron current moves through each wire in opposite directions (say, up along the right wire and down along left wire - this is position B of DPDT switch), the two wires will repel and push away from each other. The attraction and repulsion easily can be observed near center of the wires where sideways movement is the largest.

4. Illustrate attraction and repulsion between the two wires by throwing the DPDT switch first to position A, then to position B. BE SURE NOT TO LEAVE DPDT SWITCH IN POSITION A OR B FOR MORE THAN SEVERAL SECONDS, SINCE VERY LARGE CURRENTS ARE DRAWN. LARGE CURRENTS DRAWN FOR EVEN A FEW MINUTES WILL DRAIN BATTERY OR OVERHEAT WIRES.

5. Refer to Figure 6d; it shows direction of a magnetic field set up around a straight, current-carrying wire. Refer to Figures 8 and 10, which show how to determine the direction of force on a wire carrying a current when that wire is in the presence of a magnetic field. The wire on the right side in Figure 12a creates a magnetic field around the wire on the left side. After determining which way the electron current moves in the left wire for either position A or B of the DPDT switch, deduce (with the help of Figures 6d, 8 and 10) the direction of force on left wire, thereby verifying attractive or repulsive forces observed in the actual experiment.

MAKING A LIFTING ELECTROMAGNET

MATERIALS

20 x 1 cm bar of soft iron bent into U-shape. (See Figure 13.)
Several 1 1/2-volt laboratory dry cells.
About 125 ft of #25 cooper wire, single-layer, cotton-covered.
Armature bar with ring (see Figure 13).
2 plastic or wooden spools, 6 cm long x 1 cm bore x 1.5 cm wrapping radius.

PROCEDURE

Refer to Figure 13. The purpose of this experiment is to construct an electromagnet as shown in Figure 13 and to determine it lifting force.

1. Bend a piece of soft iron (20 cm long x 1 cm diameter) into a symmetrical U shape. Obtain or make two spools (6 cm long x 1 cm bore x 1.5 cm wrapping radius) that fit snugly over the ends of the U-shaped iron core.

2. Wrap the spools with #25 copper wire (single-layer cloth-cover), laying down about 45 turns per linear inch of spool. Wrap

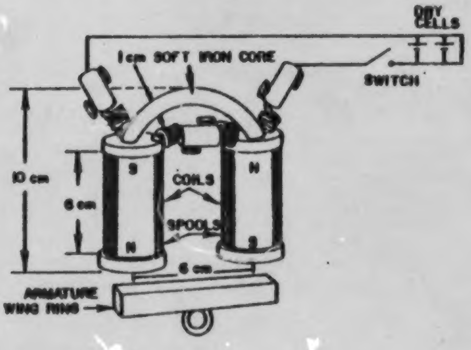


FIG. 13

each spool with two layers. Pay close attention to the direction of wrap around each coil, marking the direction, if necessary, on each coil. Slip spools on iron core, and connect two coils in such a manner that when current flows through the two spools, the ends of the spools near the ends of the iron core will be opposite poles, either N-S or S-N. Use the rule given in connection with Figure 6e to confirm that the poles are correct. For example, if the lower end of the left spool has a N pole because of direction of electron current flow through its windings, as shown in Figure 13, then the right spool must be placed on the iron core in such a manner that electron current through its windings will assure a S pole on its lower end.

3. Now connect a 1.5-V dry cell battery (or two 1.5-V dry cell batteries in parallel) across loose ends of wires on the two spools, as shown in Figure 13. Close the single-throw switch, and confirm that armature is pulled against faces of iron core spools. (If there is no magnetic attraction, carefully check the circuit and especially the spool connections.) If there is magnetic attraction, measure the lifting force of magnet by hanging known weights on the armature ring until the armature just breaks away. (Under the conditions given for construction of this electromagnet, there should not be much more than 0.5 amperes flowing through the dry cells and the wire. Nevertheless, check for overheating of spool wires; and do not leave the switch closed too long, since unnecessary battery drain will result.)

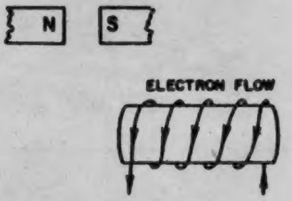
4. Discuss several possible applications of electromagnets.

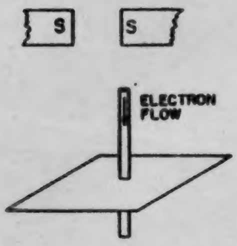
SELF CHECK

1. Explain with the use of diagrams how magnetic fields are produced by moving electrons in a permanent magnet.

2. State the rules of attraction and repulsion between north and south magnetic poles.

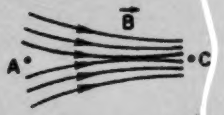
3. Sketch the magnetic-field lines in each of the following figures:

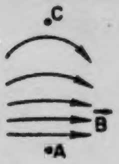




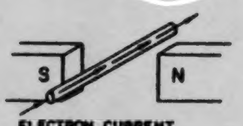
4. Indicate locations of north and south poles of solenoid in above figure.

5. An electron moves with constant speed from point A towards point C in the magnetic fields sketched below. In each case does the magnetic force on the electron increase or decrease along its path? In which direction is the electron deflected?



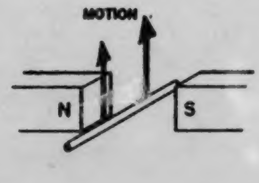


6. A long wire carrying an electron current, as shown in sketch, is located between a N-S pole pair. What is the direction of the magnetic force on the wire?



ELECTRON CURRENT

7. A long, straight conducting wire is forced upwards in the magnetic field between the N-S pair shown below. What is the direction of electron current flow in the conductor caused by the magnetic force?



8. Draw and label a diagram of an electric motor-generator.

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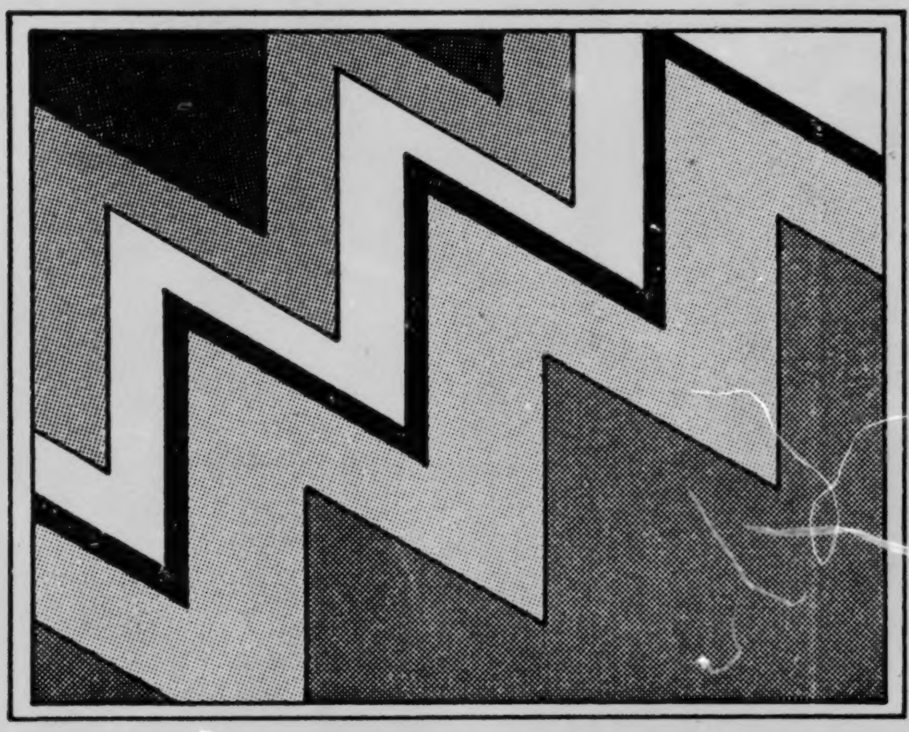
UNIFIED TECHNICAL CONCEPTS

MODULE 1 E 5

LEAD-ACID STORAGE BATTERY



ELECTRICAL APPLICATION OF FORCE

Technical Education Research Center – Southwest TERC - SW

INTRODUCTION

The voltage of a lead-acid battery is created by the presence of a potential difference between its positive (+) and negative (-) terminals. Most lead-acid automobile batteries consist of six cells, each with a potential difference of about 2 volts. As all six cells are connected in series, the potential difference for the battery approaches 12 volts.

During charging and discharging, the voltages of each cell vary and are affected by external and internal conditions of the battery. This module will describe the basic mechanisms that cause a lead-acid battery to operate and the forcelike quantity produced as potential difference is altered. Some of the significant do's and don'ts of battery care, use and maintenance will be described.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force."

OBJECTIVES

To complete this module, the student should be able to:

- List and describe in one or two sentences the five basic parts of a lead-acid storage battery.
- Name two chemicals being produced during discharging of a battery; one is deposited on the grids; the other affects electrolyte concentration (specific gravity).

- Briefly describe the rating mechanisms used to classify batteries.
- Given a temperature such as 0° F, 32° F, or 80° F, identify the appropriate percentage of available power from a fully charged battery.
- Identify the appropriate normal specific gravity for a charged battery and the instrument used to measure specific gravity.
- List three circumstances in which a fast charge rate should not be used.
- In one sentence each, describe four indications that can be used to identify a sulfated battery.
- Discuss in two or three sentences the reason for removing the caps of a battery during charging and potential effects if they are not removed.
- List at least one reason for charging a battery only in a well ventilated area.

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BATTERY CONSTRUCTION

Lead-acid storage batteries used to run electric motors or start internal combustion (gasoline or diesel) engines in most motorized vehicles consist of five basic parts:

Case.
Vented caps.
Postive and negative grids.
An electrolyte.
Positive and negative terminals.

Each part has specific functions and a set of design requirements.

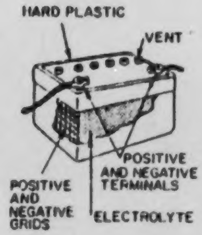

Fig. 1

The outer case must be made of noncorrosive hard rubber or plastic, preferably seamless to prevent possible leakage. Often, the top has a cap (through which fluids are added) covering each cell. Each cap has a small vent hole in it, which allows the release of internal gases (Figure 1). Each cell is separated by a partition to prevent slopping of the fluid in the battery during stops, starts and turns.

The positive grid is a metal plate of lead peroxide that accumulates a positive (+) charge in the battery. Within that same cell is another lead grid, made of lead oxide and called spongy lead, which acts as an accumulator of negative (-) charges (Figure 2). These grids are separated by noncorrosive partitions of plastic of glass wool, preventing buildup of lead sulfate on the negative grid that could form deposits called "lead trees" (because they resemble miniature tree branches) touching the positive grid and shorting the cell (Figure 3).

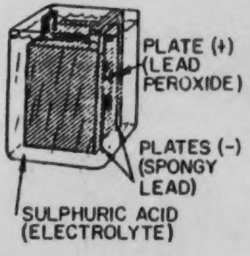

Fig. 2

Both positive and negative grids are surrounded by a concentrated solution of sulfuric acid. Sulfuric acid is the electrolyte used in most lead-acid batteries and acts as a medium for movement of the positively and negatively charged particles. This acid is extremely corrosive, and care should be used not to get it on skin or in the eyes. An immediate water flush should be done if either situation occurs.

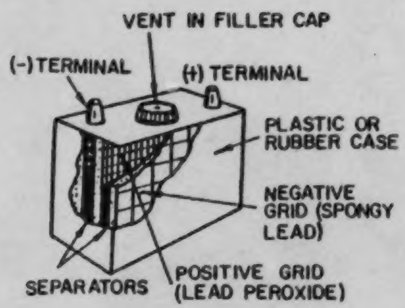

Fig. 3

The terminals shown connecting the grids in Figure 3 are different in size, with the positive terminal being larger than the negative one to help eliminate reversing connections during installation.

DISCHARGING

Production of + and - charges occurs at the respective grids, even when a battery is not in use. Sulfuric acid breaks down the lead peroxide, and positively charged particles accumulate on the positive grid until there is room for no more particles (Figure 4).

A similar process produces negative charges until the negative grid can hold no more negative particles, which slows the chemical reaction until it almost stops. When these conditions exist, maximum potential difference for the battery is achieved. For a 12-volt (a forcelike quantity) automobile battery in good condition, this should be about 12.6 volts. If the terminals are connected by a meter or any mechanism that will allow accumulated charges to move through the system, discharging will occur (Figure 5).

As particles leave the terminals, the chemical reactions in the cell resume and more charged particles are deposited at the terminals.

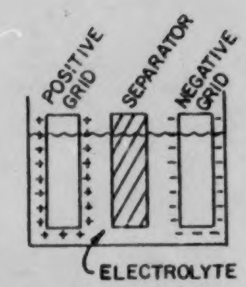

Fig. 4

Deposits of a new substance, lead sulfate, occur on each terminal and decrease chemical activity. Also during discharge, the sulfuric acid is being diluted (weakened) by newly created water molecules. If allowed to continue discharging, the battery will decrease voltage produced until it is completely discharged. When allowed to remain in a discharged state for long periods, the lead sulfate will form hard crystals and decrease the usefulness of the battery. If left discharged during cold weather, the increase in water content of the electrolyte also makes it more susceptible to damage by freezing.

Voltage capacity of a lead-acid battery is designated by a forcelike quantity determined by the manufacturer and an ampere-hour rating is assigned. The test is made by discharging at a constant rate for 20 hours and measuring the average volts per cell (must be 1.75 volts or above). If, for instance, the battery supplies 3 amps under these conditions as a result of Equation 1,

$$\text{amps} \times \text{hours} = \text{amp/h}$$

$$3 \text{ amperes} \times 20 \text{ hours} = 60 \text{ amp/h}, \qquad (1)$$

it is rated as a 60 ampere-hour battery.

Fig. 5

Other rating systems use the starting amp needs of a vehicle for a given period to classify their batteries (usually 350-500 amps). The newer system gives a maximum ampere discharge during a thirty-second interval, and a good battery will have a 375-ampere rating or greater.

Temperature has a strong effect on discharge rate. Chemical reactions and therefore the production of + and - charges are slowed by low temperatures. In cold climates battery warmers using household current maintain a minimum temperature for proper battery operation. Figure 6 summarizes effects of temperature on a fully charged battery at 0°, 32° and 80° F.

Acid-water ratio is determined by measuring specific gravity of the battery fluid. Water has a specific gravity of 1.000, and the more concentrated the acid the greater the specific gravity. Fully charged batteries have an acceptable specific gravity range of 1.200 to 1.300, with 1.260 being a suitable normal level. Table 1 shows freezing points for electrolytes at various specific gravities.

TABLE 1: ELECTROLYTE SPECIFIC GRAVITY AND FREEZING POINTS

Specific Gravity	Freezing Point, °F
1.180	-6
1.200	-16
1.220	-31
1.240	-51
1.260	-75
1.280	-90
1.300	-95

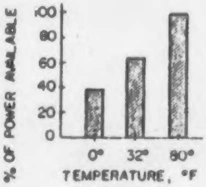

Fig. 6

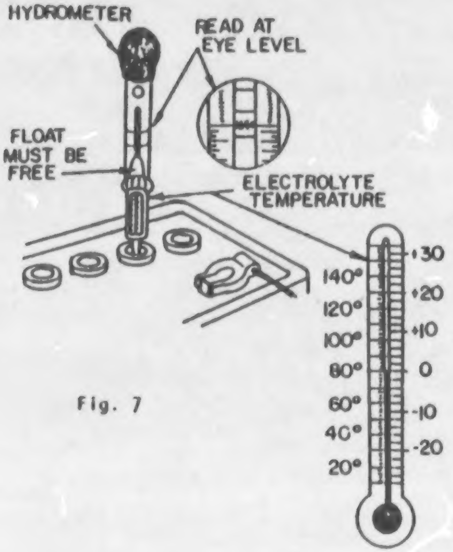

Fig. 7

A hydrometer (Figure 7) is often used to measure specific gravity. Most hydrometers will have a built-in thermometer and temperature correction chart to allow readings at various temperatures. The expanded thermometer in Figure 7 indicates the number values to be added to or subtracted from the float reading (1250 in Figure 7). In this case (the example shown in Figure 7), no correction need be added to or subtracted from the float reading at 80° F.

High temperatures increase chemical activity and water loss from the battery. Fluid levels should be checked regularly and water added when necessary. Severe water loss results in acid concentration which may cause internal damage.

If a battery is stored for an extended period (months), it will gradually lose its charge. Figure 8 gives an indication of the approximate loss after three months' storage at different temperatures.

Storing at a cool temperature will maintain the highest charge possible for the longest period of time. Never store a battery in contact with the earth; always insulate by placing it on dry boards, concrete or other nonconductors to prevent gradual grounding of the charge.

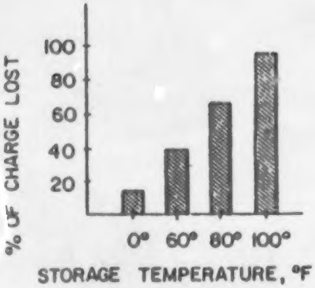

Fig. 8

CHARGING

Three aspects of charging must be reviewed:

Chemical actions occurring.
Effects of rate of charging.
By-products created.

During the charging process two main chemical changes take place as a result of forcing current in an opposite direction into the battery; the first is removal by chemical change to sulfuric acid of lead sulfate deposited on the positive and negative grids. Second, to complete the conversion, increased water created during discharge is reduced chemically to help make sulfuric acid. Two chemical reactions are given in Equations 2 and 3.

At positive grid

$$\begin{array}{ccccccccc} \left(\begin{array}{c}\text{lead}\\ \text{sulfate}\end{array}\right) & + & (\text{water}) & + & (\text{oxygen}) & \rightarrow & \left(\begin{array}{c}\text{lead}\\ \text{peroxide}\end{array}\right) & + & \left(\begin{array}{c}\text{sulfuric}\\ \text{acid}\end{array}\right) \\ \hline PbSO_4 & + & H_2O & + & O & \rightarrow & PbO_2 & + & H_2SO_4 \end{array} \qquad (2)$$

At negative grid

$$\begin{array}{ccccccc} \left(\begin{array}{c}\text{lead}\\ \text{sulfate}\end{array}\right) & + & (\text{hydrogen}) & \rightarrow & (\text{spongy lead}) & + & (\text{sulfuric acid}) \\ \hline PbSO_4 & + & 2H & \rightarrow & Pb & + & H_2SO_4 \end{array} \qquad (3)$$

The arrow in each statement (equation) indicates that the chemicals on the left react or combine to produce the chemical on the right. The direction the arrow is pointing can be reversed for a look at the discharging process.

Depending on condition of the battery and its service needs, either fast charging or slow charging can be used. Often the fast charge method (30 to 90 minutes) is used when a battery is to be left in a system and is needed for immediate use. There are some limitations to and undesirable side effects from fast charging and as a rule, do not attempt to fast charge a battery that is within 75% of its charged state. Table 2 gives an indication of the approximate specific gravities and relative state of charge of a battery. Temperature rise (up to 125° F or more) can be rapid, and severe internal damage will occur.

Fast charging also increases production of hydrogen and oxygen gases, which bubble off and change the potential specific gravity of the electrolyte. If fast charging continues and a battery has badly sulfated positive and negative grids, sulfated lead will flake off and in settling to the bottom can short out the cell. Once on the bottom it cannot be converted back to a usuable electrolyte, thus altering specific gravity of the battery. Presence of the sulfated lead also resists the current flow from the charger,

TABLE 2: CHARGE LEVEL OF A BATTERY AT VARIOUS SPECIFIC GRAVITIES

Specific Gravity	State of Charge
1.260-1.280	100%
1.230-1.250	75%
1.200-1.220	50%
1.170-1.190	25%
1.110-1.160	Very low state of charge to completely discharged.

resulting in an increase in temperature, thereby compounding the problem.

Fast charging a normal, healthy battery (assuming close supervision of temperature) can, however, quickly restore 70 to 80% of its electrical potential. During a fast charge, specific gravity should be repeatedly checked; as it approaches that of a fully charged battery the rate of charge should be tapered off. Since temperature of the electrolyte will change during charging, make sure the hydrometer being used has a temperature-measuring device and a correction-for-temperature chart to obtain an accurate specific gravity reading.

If the battery has hardened + and - grids as a result of standing in a half charged or fully discharged condition for long periods, do not use a fast charge; a slow charge should be used. Four indicators can be used to recognize batteries with lead-sulfated positive and negative grids.

Battery temperature will increase quickly during the charging process.
Large amounts of gas will be released, even at a normal charge rate.
Specific gravity will increase very slowly or not at all during charging.
Extreme voltage will be needed to achieve a normal charging rate.

If these conditions show sulfated grids, use a lower charging rate and a longer time interval. A badly sulfated battery may require as long as 100 hours to charge at 1 to 2% of its ampere-hour rating. When none of these conditions exists, a normal slow charge rate of about 5% of the ampere-hour rating for a 12- to 24-hour period is advised.

When charging a battery, it is always good practice to remove the caps covering the cells and charge in a well ventilated area, away from spark-producing machinery. The gases produced (hydrogen and oxygen) are very explosive, and care should be exercised not to create a spark near the battery that could also result in an explosion, scattering the highly dangerous acid.

MATERIALS

Two lead strips.
Low-voltage power supply.
Container with clips for lead strips.
Enough sulfuric acid (H_2SO_4) to fill container about 3/4 full. Solution should be about 8:1 in concentration.
One pad of steel wool.
Multimeter.
One set of leads with alligator clips.
One set of leads for power supply.
One 6-volt lamp with socket.

PROCEDURES

1. Clean both strips until they shine by polishing with steel wool.

2. Attach strips to retainer clips on container.

3. Carefully pour sulfuric acid into container until about 3/4 full.

   CAUTION: This acid solution is extremely dangerous. Do not spill or splash while filling container. A rubber apron, rubber gloves and eye protection should be worn when handling this material. Severe burns will result if it is allowed to contact skin. Quickly flush with water anything mistakenly touched by acid solution.

4. Connect voltmeter leads to retainer clips.

5. Connect power supply (in off position) to retainer clips; check to be sure positive lead from voltmeter is connected to positive lead from power supply.

6. Touch one lead from bulb to each retainer clip; note reaction.

7. Turn on power supply; adjust voltage to 3 or 4 volts.

8. Note reaction and any color change at each terminal.

9. After about 7 to 8 minutes, turn off power supply; read voltmeter; record value here. __________.

10. Connect bulb leads to terminal clips; note reaction of bulb and voltmeter.

11. Leave bulb connected to terminals until it goes out; note any color or physical change that occurs at terminals.

12. Check with instructor to ensure observations are consistent with predetermined results.

SELF CHECK

1. White a one or two sentence description of each of the five basic parts of a lead-acid storage battery.

2. Identify the two chemicals produced on the cells during discharging of a battery.

3. In one or two sentences describe two rating systems used to classify batteries.

4. What percent of the rated power of a fully charged battery will be available when temperature is 32° F?

5. What instrument is used to measure specific gravity, and what is the normal expected value of specific gravity for a fully charged battery?

6. List three conditions under which a fast charge should not be applied to a battery.

7. List four ways to recognize a sulfated battery.

8. Why are the caps of a battery removed during charging, and what may happen if they are not?

9. Why must a battery be charged in a well ventilated area?

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See especially pp. 285-292.

1. This procedure can be dangerous if materials are not handled properly; close supervision is suggested.

2. No reaction should result by touching leads to retainer clips before charging in step 6.

3. At step 8, the positive terminal should become dark gray or black (lead peroxide), and the negative terminal will remain shiny.

4. Also note that large bubbles are produced at the negative terminal and smaller ones at the positive terminal.

5. In step 9, voltage recorded will vary, but it probably will be about 2.0 volts ± 0.5 volts.

6. Bulb should light at step 10.

7. If students are observant at step 11, they will note the bulb glows for about 30 seconds and starts to dim. It will go out at the same instant the large bubbles are released by the negative plate.

8. This procedure can be repeated as often as desired with other items such as door buzzers substituted for the light bulb.

9. The charging voltage may be varied, but little or no difference will be noted beyond 6 volts.

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UNIFIED TECHNICAL CONCEPTS

MODULE 1 T 1

THERMOMETERS

THERMAL APPLICATION OF FORCE

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INTRODUCTION

The thermometer is an instrument used to measure temperature. A portion of the thermometer is brought into thermal equilibrium with the body whose temperature is being measured. The indicated temperature is the equilibrium temperature of the sample and the thermometer sensor. For example, an ordinary medical thermometer consists of a bulb filled with mercury that expands into a tube as its temperature rises. This force of expansion produces a temperature reading as the mercury level indicates the temperature of the bulb and the patient on the calibrated tube. There are other types of thermometers, several of which are included in this module. In the laboratory, the student will calibrate thermometers and convert from Fahrenheit to Celsius scales.

PREREQUISITES

Student should have completed Concept Module I-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the terms:

  Fahrenheit temperature scale.
  Celsius temperature scale.
  Absolute zero.
  Kelvin temperature scale.
  Rankine temperature scale.

- Draw the following devices and label the basic parts:

  Thermocouple.
  Pyrometer.
  Liquid-in-glass (thermometer).

- Convert a temperature given in one scale to its equivalent in any other scale without being given the conversion formula.

- Given two liquid-in-glass uncalibrated thermometers, calibrate one in Celsius and one in Fahrenheit and measure a given container of water within ±2°.

DISCUSSION

The most commonly-used type of thermometer is the liquid-in-glass thermometer, illustrated in Figure 1.

Volume is one physical property of a liquid that changes with temperature. The volume of mercury, for example, increases if the temperature is increased and decreases if the temperature is decreased. The increased volume of warmed mercury within the confined glass tube of the thermometer causes the level to rise. By selecting certain reference points (such as the freezing and boiling point of water) and uniformly subdividing the distance between them, a temperature scale is established.

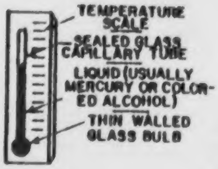

TEMPERATURE SCALE
SEALED GLASS CAPILLARY TUBE
LIQUID (USUALLY MERCURY OR COLORED ALCOHOL)
THIN WALLED GLASS BULB

FIG. 1

TEMPERATURE SCALES

Until recently the "Fahrenheit" temperature scale was used almost exclusively in the United States. There is now an intensive effort to adopt the "Centigrade," or "Celsius," scales used throughout the rest of the world. In defining either scale, it is necessary to determine two reference-point temperatures. Once these reference points are determined, the position of the zero point and the size of the temperature unit is obtained. One of these reference points is called "freezing point." This is the temperature at which water changes to ice. The other point is called the "boiling point." This is the temperature at which the water boils and changes to steam. On the Celsius scale, the freezing point is given a value of 0° and the boiling point, 100°. On the Fahrenheit scale these points are assigned the numbers 32° and 212° respectively.

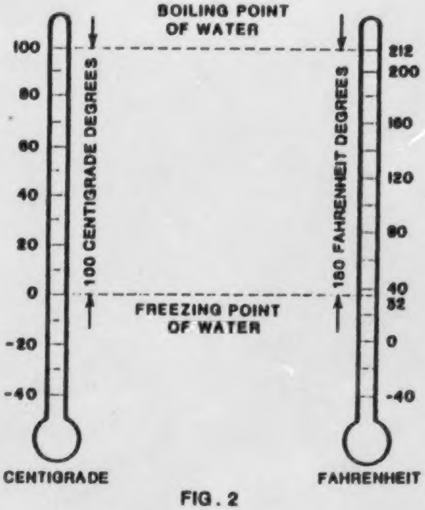

BOILING POINT OF WATER
100
80
60
40
20
0
-20
-40
100 CENTIGRADE DEGREES
180 FAHRENHEIT DEGREES
212
200
160
120
80
40
32
0
-40
FREEZING POINT OF WATER
CENTIGRADE
FAHRENHEIT

FIG. 2

When constructing a liquid-in-glass thermometer with a Celsius scale, first insert the thermometer in an ice-water mixture and carefully mark the tube at the height of the liquid column. Then immerse the thermometer in boiling water and again mark the height of the liquid column on the tube. The distance between these marks is then divided into one hundred equal parts and numbered from 0° to 100° as shown in Figure 2. If required, the scale can be extended above 100° and below 0°. The Fahrenheit scale, illustrated in Figure 2, is determined in a similar way by dividing the length of the column between the freezing and boiling points into 180 equal divisions and extending the scale in either direction. 0°C is the equivalent temperature to 32°F and 100°C is the equivalent temperature to 212°F.

There is a simple method of recalling the relationship between Fahrenheit and Celsius temperature scales. If the temperature in Fahrenheit is known and is to be converted to Celsius, simply draw two simple thermometers and establish ratios as shown in Example A.

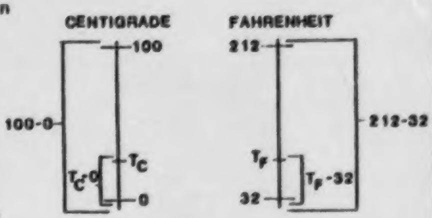
EXAMPLE A: SCALE RATIO BETWEEN CENTIGRADE AND FAHRENHEIT

Given: T_C or T_F.

Find: Their equivalent in the other system.

Solution: Equate the ratios from both sides:

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32}$$



Solving this ratio for T_C or T_F yields:

$$T_C = \frac{5}{9}(T_F - 32)$$

$$T_F = \frac{9}{5}(T_C + 32)$$

Example B shows a calculation for the temperature values when T(°F) = T(°C).

EXAMPLE B: CALCULATION OF VALUES WHEN T°F = T°C

Given: T(°F) = T(°C).

Find: The actual temperatures at this condition.

Solution:

$$T_C = \frac{5}{9}(T_F - 32)$$

$$T_C = \frac{5}{9}(T_C - 32)$$

$$9\ T_C = 5\ T_C - 160$$

$$4\ T_C = -610$$

$$T_C = -40° = T_F$$

At -40° the temperature scales are the same, i.e., -40°C = -40°F.

ABSOLUTE TEMPERATURE SCALES

There is an experimental and theoretical lowest temperature known as "absolute zero." Absolute zero is -273.2°C or rounded off, -273°C. Since it is sometimes inconvenient to work with negative temperatures, temperature scales have been defined whose zero point is absolute zero. These scales are called "absolute temperature scales." Sales using the Fahrenheit degree and the Celsius degree as their smallest divisions are both in common use and are illustrated in Figure 3. The scale using the Celsius degree is called the "kelvin" scale, and the scale using the Fahrenheit degree is called the "rankine" scale.

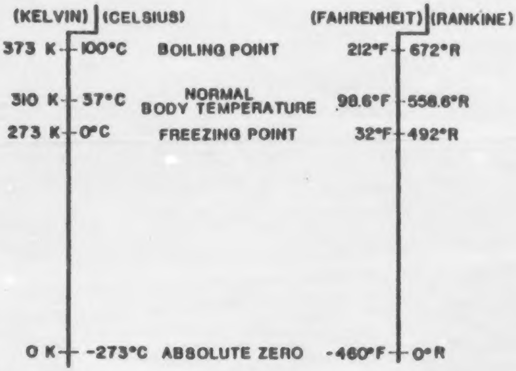

(KELVIN)
(CELSIUS)
(FAHRENHEIT)
(RANKINE)
373 K
100°C
BOILING POINT
212°F
672°R
310 K
37°C
NORMAL BODY TEMPERATURE
98.6°F
558.6°R
273 K
0°C
FREEZING POINT
32°F
492°R
0 K
-273°C
ABSOLUTE ZERO
-460°F
0°R

FIG. 3

Absolute temperatures can be obtained with the aid of Equations 1 and 2.

$$T(K) = T\ (°C) + 273 \qquad (1)$$

$$T(°R) = T\ (°F) + 460 \qquad (2)$$

Note: The symbol for degrees Kelvin in the SI systems of units is K and does not include the degree (°) symbol.

OTHER THERMOMETERS

The thermocouple shown in Figure 4 works on the principle that when wires of any two unlike metals are formed into a complete circuit (as shown in Figure 4), an emf is generated whenever the junctions (A and B) and the junction C are at different temperatures. In the circuit shown, the junctions A and B are kept at 0°C by ice and water in a Dewar flask (essentially a thermos bottle). The

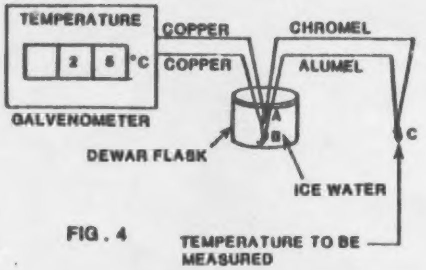


FIG. 4

junction C is placed in contact with the body whose temperature is to be measured. Since these junctions will be at different temperatures, a voltage is generated, read by the galvanometer G, and converted into a temperature read out.

PYROMETER

The optical pyrometer shown in Figure 5 is used primarily to measure hot temperatures from a distance, accomplished by looking at a hot furnace through a special telescope containing a lamp filament and a red filter A. By adjusting the rheostat D the current through the lamp filament B is increased and the filament starts to glow. The current through the filament is increased until the color of the filament just matches the background color of the oven. The temperature of the oven then can be read on meter E.

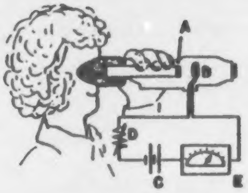


FIG. 5

SUMMARY

The most common thermometer is a device that measures temperatures by gaging the amount of expansion of mercury within a glass tube. Heat causes a thermal-expansion force used for measurement purposes. Other thermometers include the optical pryometer and the thermocouple used with a galvanometer.

THERMOMETER CALIBRATION

MATERIALS

Celsius thermometer.
Fahrenheit thermometer.
Ice and water and two beakers.
Source of heat (Bunsen burner).
Ring stand and ring.
Wire gauze.
Roll of duct tape or equivalent.
Masking tape.

PROCEDURE

1. Select one Fahrenheit and one Celsius thermometer.

2. Cover etched numbers on the thermometer with a tape, leaving a portion of glass uncovered to observe mercury or alcohol tube (capillary).

3. Pack crushed ice and water in a beaker; and mark lowest level of mercury on both tubes, as shown in Figure 6. Record in Data Table.

4. Place thermometers in beaker of water at room temperature.

5. Slowly bring water to a boil. Mark highest point to which mercury rises, as shown in Figure 7. Record.

6. Allow thermometers to cool. On one thermometer, divide space between freezing and boiling into 100 divisions, and on the other tube into 180 divisions. A strip of masking tape can be attached to the tube, and intervals can be marked on tape. Mark every 5 divisions or so. The first thermometer is calibrated in Celsius degrees; the second, in Fahrenheit degrees.

7. Place both thermometers in a container of water at room temperature, and record reading on each scale in Data Table.

8. Heat water slowly until Fahrenheit thermometer reads 170°F. Record the temperature of Celsius thermometer.

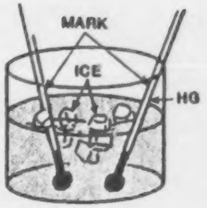


FIG. 6

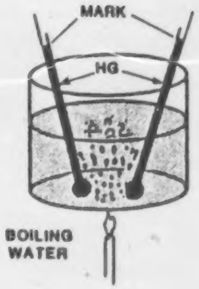


FIG.7

9. Convert the "°F" temperature to "°C" by using equation in Data Table.

10. Compare it to actual "T_C" reading in Step 8.

DATA TABLE					
Type Thermometer	Ice Water (Lowest Level)	Boiling Water (Highest Level)	Room Temp.	Temp. of Celsius Thermometer at 170°F	Temp. Conversion °F to °C
Celsius (100 divisions)			°C		(see below equation)
Fahrenheit (180 divisions)			°F	170°F	

Equation for Converting °F temperature to °C.

$$T_C = \frac{5}{9}\ (T_F - 32) = \frac{5}{9}\ (170^\circ C - 32^\circ C)$$

SELF CHECK

1. Define the following terms:

   Fahrenheit temperature scale.
   Celsius temperature scale.
   Absolute zero.
   Kelvin temperature scale.
   Rankine temperature scale.

2. Describe and illustrate with schematics the basic operation of a:

   Thermocouple.
   Pyrometer.
   Liquid-in-glass (thermometer).

3. The melting point of gold is 1063°C. What is its melting point in degrees Fahrenheit?

4. Normal room temperature is 72°F. What is room temperature in °C?

5. Convert each of the following to °C

   a. -32°F.
   b. 350 K.
   c. 28°F.
   d. 1300 K.

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UNIFIED TECHNICAL CONCEPTS

MODULE 1 T 2

TEMPERATURE DIFFERENCE AND HEAT FLOW

THERMAL APPLICATION OF FORCE

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INTRODUCTION

The efficient movement, or transmission, of heat energy from one region to another is an important technology. From early times, people have had to seek ways to protect themselves from the intense heat and cold of the environment. With the development of civilization, problems involving heat transmission have become more complex.

In this module, the student will be introduced to the three ways by which heat is transmitted: conduction, convection and radiation, and will see that temperature difference causes the movement of heat energy, and acts, therefore, as the forcelike quantity in the thermal system. In the laboratory, the student will determine the conduction of heat through a common material.

PREREQUISITES

Student should have completed Concept Module 1-0, "Force."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define these terms:

  Heat conduction.
  Heat convection.
  Heat radiation.
  Thermal equilibrium.

- Calculate heat flow rate, given formulas and material properties.
- Measure, record, and graphically represent temperatures taken at equal time intervals for various positions within a slab of conducting material; and explain the characteristics of the curves obtained.

If one end of a metal rod is placed in a flame and the other end is held in the hand, that part of the rod being held soon feels warm, although the end being held is not itself in direct contact with the flame. In this case, heat reaches the cooler end of the rod by conduction. Heat is conducted through the metal rod from the warm end to the cool end.

Heat also can be transferred from one place to another by the actual motion of hot gases or liquids. Hot air rising over an open campfire or a candle is an example. Steam rising over a pot of boiling water shows how hot air moves, circulating heat throughout the room. Heat moved in this manner is due to a process called "convection." If the heated gas or liquid is forced to move by a fan or blower, the process is called "forced convection." An automobile uses forced convection to move heated water from engine to radiator and back. The hot water is cooled while passing through the radiator by forced convection of air provided by the fan and movement of the vehicle along the road.

A third way in which heat can be transmitted is by radiation. When one stands in front of a fireplace, heat is radiated through space from the fire to the person's body in the form of invisible infrared light. In this way, heat from the sun reaches the earth.

Whether heat is transferred by conduction, by natural convection, or by radiation, it always flows spontaneously from a region of high temperature to a region of low temperature. The larger the temperature difference between source and receiver, the more easily heat is moved. Temperature difference is the forcelike quantity that causes movement of heat energy in the thermal system.

On very hot days, an air conditioner removes heat that leaks into a building. On cold days, a furnace replaces heat that leaks out of the building.

Movement of heat in a building is similar to movement of water in a leaking ship. If a ship leaks, the bilge pump is turned on. If water is leaking faster than the bilge pump can remove it, the ship eventually will fill with water and sink. On the other hand, if the leak is slow enough, the bilge pump easily can remove the water.

When a building is designed, its "heat losses" and "heat gains" must be considered. Heating and cooling loads must be based on heat loss and heat gain. Engineers and architects can control, to a degree, heat losses and gains by choosing the proper materials to be used in constructing and insulating houses.

To show that heat flows from a region of high temperature to a region of low temperature, consider the situation illustrated

In Figure 1. Figure 1 depicts a room maintained at a temperature of 75°F by an attic heater. The heater pumps heated air into the room through ducts located in the ceiling (1). Outside temperature is 32°F. Heat from the interior of the house "leaks" to the outside through many different paths. Some heat is conducted through windows (2 and 3), while some is conducted directly through the insulated wall (4). Other conduction paths are through the brick wall (5) and the concrete slab (6). Much heat escapes through the ceiling (7) and through the roof (8). No matter which path the heat follows, it always moves from the warmer area (room) to the cooler area (outside).

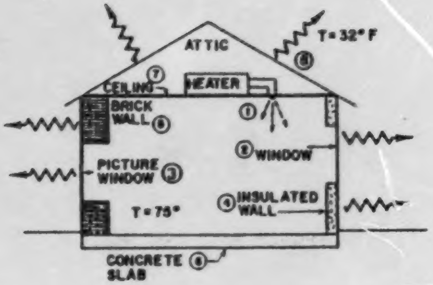

FIG. 1

When temperature difference between the outside and inside of the house shown in Figure 1 is not large, heat losses will be small; and the heater will not be required to run continuously. On the other hand, if the outside temperature should drop to -40°F, the heat loss may be greater than heat provided by the heater. If the heater were turned off, the inside of the house would eventually reach outside temperature; and, at that point, temperature difference would equal to zero. When this occurs, the house and its environment would be in thermal equilibrium; and no heat would flow from one to the other.

In industries, equipment gets so hot that water or other liquids are required to flow over or around them to carry away heat. Examples are automobile engines, laser heads, nuclear reactors, cutting tools, and some machines and electrical components. In all of these examples, heat is transferred from the hot objects to the cooler fluids by convection, by conduction, or by radiation.

CONDUCTION EXPERIMENT

Different materials conduct heat at different rates, as can be demonstrated by touching iron and wood (at room temperature) with a hand (at body temperature). The iron is considerably cooler than the wood. This is because heat is conducted away from the warmer hand much more quickly by iron than by wood. Poor conductors (wood, felt, air, asbestos) are called "thermal insulators." The ability of a substance to transmit heat by conduction is a physical property of the particular material. Heat transferred by conduction through a material (Figure 2) can be calculated with Equation 1.

$$Q_H = \frac{kA\ (T_2 - T_1)}{d} \qquad (1)$$

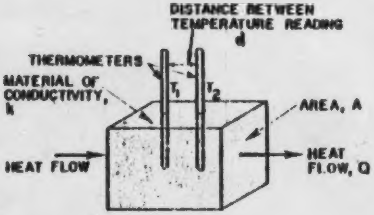
DISTANCE BETWEEN TEMPERATURE READING
d
THERMOMETERS
MATERIAL OF CONDUCTIVITY, k
T_1
T_2
AREA, A
HEAT FLOW
HEAT FLOW, Q

FIG. 2

where

Q_H = heat flow rate, Btu/h or cal/s.
k = coefficient of thermal conductivity,

$$\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2\text{F}^\circ} \quad \text{or}$$

$$\frac{\text{cal} \cdot \text{cm}}{\text{h} \cdot \text{cm}^2\text{C}^\circ}$$

A = area ft^2 or cm^2.
$(T_2 - T_1)$ = difference in temperature between the two surfaces "d" ft or cm apart.
d = thickness of two surfaces considered, ft or cm.

Example A illustrates the use of this equation in solving a problem.

EXAMPLE A: HEAT FLOW THROUGH INSULATION

Given: An 80 ft wall is insulated with 3 in of glass wool having a thermal conductivity of 0.29 (Btu · in)/(h · ft^2 · F°). The inside temperature is 72°F, and the outside temperature is 18°F.

Find: The heat energy lost through the wall in 24 hours. (Neglect the insulating effects of other materials in the wall.)

Solution:

$$Q_H = \frac{kA\ (T_2 - T_1)}{d}$$

$$= \frac{\left(0.29\ \frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot \text{F}^\circ}\right)\ (80\ \text{ft}^2)(72^\circ\text{F} - 18^\circ\text{F})}{(3\ \text{in})}$$

$$= \left(0.29\ \frac{\text{Btu} \cdot \cancel{\text{in}}}{\text{h} \cdot \cancel{\text{ft}^2} \cdot \cancel{\text{F}^\circ}}\right)\left(\frac{80\ \cancel{\text{ft}^2}}{3\ \cancel{\text{in}}}\right)\ (54^\circ\cancel{\text{F}})$$

$$Q_H = 417.6\ \text{Btu/h}$$

Heat energy escapes through the wall at a rate of 417.6 Btu/h. Thus, in 24 hours the heat loss is —

H = (417.6 Btu/h)(24 h)

H = 10,022 Btu

SUMMARY

In thermal systems, the forcelike quantity is temperature difference. Heat energy flows in any system in response to the temperature differences in the system, and the rate at which heat flows through any substance depends upon the temperature differences across that substance, as well as upon the characteristics of the substance. Thermal insulating materials are chosen for their low thermal conductivity.

HEAT FLOW BY CONDUCTION

MATERIALS

Electric temperature-measuring device such as a thermocouple or a calibrated thermistor.

Slab of conducting material (concrete, brick, etc). Slab can be made from a block of solid concrete (a 2" concrete patio block) with holds drilled at 3-cm intervals across the center of the block. Five holes are needed to hold temperature sensors. Holes should be just large enough to hold the sensors snugly to reduce heat losses.

Propane torch on a solid stand.

Graph paper.

PROCEDURE

1. Set up conductive slab and source of high temperature as shown in Figure 3. Be certain the experiment is positioned well away from any flammable material.

2. When everything is set up and fire safety is ensured, record initial block temperature (T°) in Data Table. Initial temperatures will correspond to room temperature and should be the same for all sensors.

3. Apply the constant heat source to end of insulated slab; record temperature readings at each sensor position at two-minute intervals.

4. When temperature of sensor nearest the hot end approaches upper limit of its calibration,

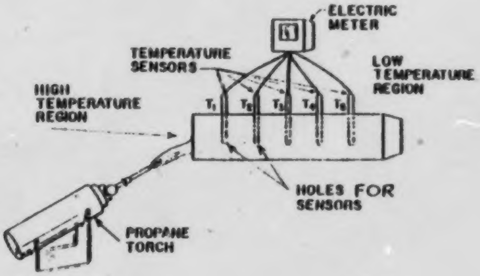
ELECTRIC METER
TEMPERATURE SENSORS
LOW TEMPERATURE REGION
HIGH TEMPERATURE REGION
T_1
T_2
T_3
T_4
T_5
HOLES FOR SENSORS
PROPANE TORCH

FIG. 3

remove heat source; and record all sensor readings in Data Table.

5. Plot data on a graph similar to Figure 4.

6. Explain what your graph shows about heat conduction through the material. How does this experiment demonstrate temperature difference as a forcelike quantity?

DATA TABLE

Thermometer Location		Time (2-minute intervals)								
		t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
1										
2										
3										
4										
5										

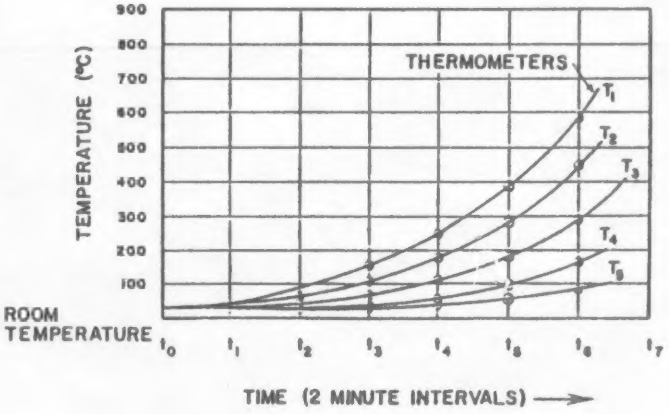

FIG. 4

SELF CHECK

1. Define the following terms:

   Heat conduction.
   Heat convection.
   Heat radiation.
   Thermal equilibrium.

2. An old window, 5 ft x 8 ft, was removed, and a wall of 4-inch face brick was laid in the opening. On a winter day, the outside surface of the brick was 18°F, and the inside surface was 58°F. What is heat flow rate in Btu/h through the bricked-up window?

$$K_{(\text{brick})} = 9.0\,\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot \text{F}^\circ}$$

3. Given an explanation of the graph of experimental results. Why do curves rise at different rates?

4. If a material with a higher conductivity had been used in the laboratory experiment, describe qualitatively how the graphs would change.

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UNIFIED TECHNICAL CONCEPTS

MODULE 1 T 3

THERMAL STRESS

THERMAL APPLICATION OF FORCE

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TERC-SW

TECHNICAL EDUCATION RESEARCH CENTER – SW
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INTRODUCTION

As the temperature of a solid increases, the solid expands slightly. Because this is true, expansion joints are needed in pipes and bridges. Heat expansion is useful for applications such as bimetal strips in thermostats. Heat expansion of liquids is also the principle by which the ordinary thermometer works.

In this module, the student will learn how to determine the coefficient of thermal expansion of materials. Student will learn how much some materials expand when they are heated, and will learn about the practical implications of thermal expansion.

PREREQUISITES

Student should have completed Concept Module 2-0, "Force."

OBJECTIVES

To complete this module, the student should be able to:

- Mathematically describe the coefficient of expansion of a solid defining each term and including the effects of the force created if the solid were not allowed to expand.
- Given the appropriate equipment, measure the thermal expansion coefficient of a metal wire.
- State three practical applications relating to expansions and contraction with temperature change in solids.

DISCUSSION

It is well known that, when a solid is heated, it expands slightly. Its physical dimensions increase in a manner proportional to the temperature change. If the temperature increases from T_o to T, length will increase slightly from L_o to L. The mathematical relationship describing thermal expansion is given by the equation:

$$(L - L_o) = L_o \alpha \, (T - T_o) \qquad (1)$$

where

L_o = original length.
T_o = original temperature.
α = coefficient of linear expansion.

The coefficient of linear expansion α is a constant that relates the amount of elongation to a given temperature rise. If we consider a bar with an original length of one "unit" subjected to a one-degree temperature change, Equation I shows that α is just the change in length that results. Thus, materials that expand more readily will have a larger value for their coefficients of thermal expansion. Though it varies from one material to another, α is the constant characteristic of a particular material. Strictly speaking, α is not exactly a constant because its value depends somewhat on original temperature. However, for ordinary values of temperature and for ordinary ranges of temperature change, one may consider α to be a constant without introducing serious error.

Table I gives some values of the coefficient of linear expansion (α) for typical solids. Note that, if the temperature is measured in °C, α has units in reciprocal Centigrade degrees. The values of α are small, typically of the order of 0.0000005 to 0.000025 parts per Centigrade degree. This means that, for a change in temperature of one Centigrade degree, a solid sample will change its physical dimensions fractionally, by only a few parts per million.

There are a few materials that have extremely low coefficients of thermal expansion. Fused quartz and invar are examples. Invar is an iron-nickel alloy with the composition adjusted so that the coefficient of thermal expansion is very low. Such materials offer greater stability in an environment of changing temperature than materials with larger coefficients of expansion.

TABLE I. PROPERTIES OF VARIOUS MATERIALS

Material	Linear Coefficient of Thermal Expansion (Per Degree C)	Modulus of Elasticity (Dyne/cm^2)
Aluminum (commercially pure)	23.5×10^{-6}	6.9×10^{11}
Brass (yellow, 65% Cu, 35% Zn)	20.3×10^{-6}	10.3×10^{11}
Copper (commercially pure)	16.5×10^{-6}	12.8×10^{11}
Glass		
Borosilicate	6.2×10^{-6}	---
Pyrex	$(3.1 - 3.5) \times 10^{-6}$	---
Soda-lime	$(7.5 - 9.2) \times 10^{-6}$	---
Invar (64% Fe, 36% Ni)	1.2×10^{-6}	14.8×10^{11}
Kovar (54% Fe, 28% Ni, 18% Co)	5.9×10^{-6}	13.8×10^{11}
Quartz, fused	0.5×10^{-6}	7.2×10^{11}
Stainless steel (17-20% Cr, 8-11% Ni)	9.6×10^{-6}	19.3×10^{11}
Titanium (commercially pure)	8.5×10^{-6}	11.6×10^{11}
Tungsten (commerically pure)	4.3×10^{-6}	34.5×10^{11}

If temperature decreases from T_0 to T, according to Equation 1, $T - T_0$ will be less than zero (negative) and obviously L will be less than L_0. In other words, a solid will contract as it cools and expand as it is heated.

Example A shows how Equation 1 is used to solve a problem.

The coefficient of expansion as defined in Example A is called the coefficient of linear expansion because it applies to the change in only one dimension (length) of a material. If a solid cube were heated, each dimension of the cube would undergo linear expansion according to Equation 1. The result

EXAMPLE A: THERMAL EXPANSION IN A LASER CAVITY

Given: A frequency-stabilized laser operating at a precisely controlled frequency. If the length of the laser changes, the frequency will shift. Suppose such a laser, 0.3 meters long, is constructed of stainless steel and has an operating temperature change of 10 C°.

Find: Change in length of the laser cavity. What would the change be if the laser structure were made of quartz?

Solution:

a. Change in length = coefficient of linear expansion x original length x change in temperature.

$$(L - L_o) = L_o\alpha\ (T - T_o)$$
$$= 0.3\ \text{m} \times 9.6 \times 10^{-6}\ \frac{1}{C^\circ}\ (10\ C^\circ)$$
$$(L - L_o) = 2.9 \times 10^{-5}\ \text{m}$$

The change in length is 0.000029 m.

Even this change in length will cause a sizable frequency shift, which could be a serious problem in applications like laser metrology.

b. If the structure had been made of fused quartz, change in length would have been:

$$(L - L_o) = L_o\alpha\ (T - T_o)$$

or

$$(L - L_o) = 0.3\ \text{m} \times 0.5 \times 10^{-6}\ \frac{1}{^\circ C}\ (10^\circ C)/L - L_o = 1.5 \times 10^{-6}\ \text{m}$$

This change in length is 0.0000015 m, about 20 times smaller than $(L - L_o)$ with stainless steel; much less frequency shifting will result as a consequence.

would be an increase in volume of the cube. The increase of volume V is given by Equation 2:

$$(V - V_o) = (3\alpha)\ V_o\ (T - T_o)$$

or

$$(V - V_o) = V_o\beta\ (T - T_o) \qquad (2)$$

where

$T - T_o$ = temperature increase.
V_o = initial volume.
$\beta = 3\alpha$ = coefficient of volume expansion.

For solids, the coefficient of volume expansion β is approximately three times the coefficient of linear expansion α. Values of β can be determined from tables of values for α, found in engineering handbooks and physics texts.

There can be very large mechanical forces associated with the expansion of solids, even though the change in dimensions is relatively small. Suppose that a metal bar whose ends are clamped rigidly is heated. Because it is firmly clamped, it is not allowed to expand. A force will be exerted on the clamping structure. The force F may be expressed by the equation:

$$F = AE\alpha\ (T - T_o) \qquad (3)$$

where

A = cross-sectional area of bar.
$T - T_o$ = temperature increase.
α = coefficient of linear expansion.
E = constant known as modulus of elasticity.

The modulus of elasticity has dimensions of force per unit area. Some values for the modulus of elasticity are given in Table 1. Example B shows how Equation 3 is used to solve for thermal stress.

Example B shows that thermal stress can be very large. In a structure that does not allow for thermal expansion, such thermal stress could be very destructive. Because a temperature range of 50° C or more is easily covered in the course of a year in typical climates, structures exposed to weather changes must have provision for expansion to accommodate the thermal stress.

The force in Equation 3 is considered a compressive force in

EXAMPLE 8: THERMAL STRESS

Given: A stainless steel rod with a cross-sectional area of $4\ \text{cm}^2$ clamped at both ends and heated through a range of 40 Centigrade degrees.

Find: Force exerted against the clamps.

Solution:

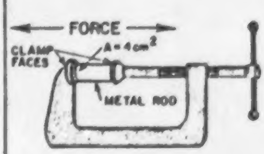
$$F = AE\alpha\ (T_1 - T)$$

$$= 4\ \text{cm}^2 \times \left(19.3 \times 10^{11}\ \frac{\text{dyne}}{\text{cm}^2}\right) \times \left(9.6 \times 10^{-6}\ \frac{1}{°\text{C}}\right) \times (40\ \text{C}°)$$

$$F = 2.96 \times 10^9\ \text{dynes}$$

The stress, (force/area) is 2.96×10^9 dynes/$4\ \text{cm}^2$.

Stress = 7.4×10^8 dynes/cm², which is equivalent to 10,732 lb/in².



the material if the material temperature increases, that is $(T - T_0)$ is positive. The force is considered to be a tensile force in the material if the material temperature decreases. In either case, the force can be sizable and must be taken into account.

There are a number of practical implications for thermal expansion and thermal stress. Some of these include:

Structures such as bridges, rails, pipes, etc. which are subject to large temperature changes, must be allowed to have parts of the structure free to slide during expansion and contraction. Pipes must also have expansion joints. Bearings in high-speed machines must have proper clearance for expansion. Bridges and steam pipes must be built with expansion structures.

If different materials are to be fastened together rigidly, they must have approximately the same coefficient of expansion. If two materials with dissimilar coefficients of expansion are sealed together and undergo a large change in temperature, the structure can be destroyed by bending. For example, in a glass-to-metal seal, kovar (an iron-

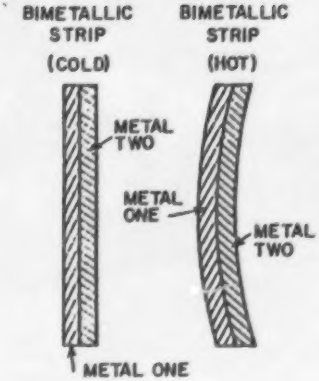


Fig. 1

nickel-colbalt alloy) is often used because its expansion coefficient is close to that of some types of glass.

Structures that require good dimensional stability are often constructed with materials having low coefficients of thermal expansion. As an example, invar and quartz are common structural materials for lasers that require good dimensional stability.

The bimetallic strip is a good example of a practical application of the expansion of materials. Figure 1 illustrates the bimetal strip. It consists of two strips of metal fastened together, so that at some temperature they are straight (drawing on the left in Figure 1). The two strips of metal have different coefficients of expansion. If the strip is heated, it will bend in a curve (drawing on the right in Figure 1). In the case shown, metal 1 has the larger value of linear coefficient of expansion. The amount of bending produced will be determined by the change in temperature experienced by the strip. Bimetallic strips are, therefore, commonly used in thermostats.

Thermometers make use of the expansion of mercury or alcohol enclosed in a glass structure. The glass structure will expand, but because the coefficient of expansion of the mercury (or alcohol) is considerably greater than that of the glass, a temperature increase causes the liquid to expand and, therefore, rise in the small bored tube of the thermometer. Thus the height of the mercury column is marked off in degrees to indicate quantitatively the change in temperature.

Telephone lines and other suspended metal cables are allowed to sag somewhat in warm weather. Otherwise, when they contract in winter, the thermal stress would snap them.

In summary, we have seen in this module how a change in temperature causes expansion of materials. It also leads to stress (mechanical compressive or tensile forces) in solids that are rigidly clamped. Where such physical changes create problems, they must be provided for in the design of physical structures. Such changes can also be put to practical use in the design of instruments like thermometers.

LABORATORY

MATERIALS

Microscope, with calibrated reticle.
Heavy wire, stainless steel (22 gage).
Clamp (vise or C-clamp).
Electrical power supply (d.c. supply with at least 10-amp capability).
Connecting wires and clamps (high-conductivity copper wires).
Pure tin foil such as Cenco #78860-003 or equivalent.
Two small hook weights approximately 10 grams and 20 grams.
Pulley to attach to edge of table.
Thermometer.
Meter stick.

PROCEDURE

In this experiment the coefficient of linear thermal expansion of a steel wire will be measured. The wire will be heated electrically and the motion of a scratch on the wire will be observed under a microscope to give a quantitative measure of the amount of motion. The experimental arrangement is shown in Figure 2.

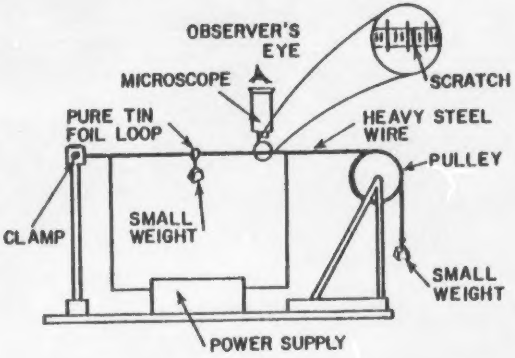


Fig. 2

1. Heavy wire should be clamped rigidly on one end with a C-clamp or a vise. Other end should be free to move over pulley as wire expands. A small weight (20 g) on end of wire will serve to keep wire straight.
2. At some distance, approximately 50 centimeters from clamping position, make a small scratch on wire. Position microscope so scratch is in field of view; observe its original position using calibrated reticle. (A length of 50 cm or so will allow a conveniently observable motion of scratch. To have reasonable characteristics for electrical heating, a wire with a diameter around 0.028 inch may be suitable - steel wire gage about 22. It may be necessary to try several wire diameters near this size to match capabilities of electrical power supply to heat wire sufficiently.)

3. To ensure that wire is at ambient temperature, measure initial temperature of environment with thermometer. With meter stick, accurately measure length of wire from position of clamp to scratch. Hang a small weight over wire from a piece of pure tin foil.
4. Connect current source to ends of wire with connecting wires and clamps. Turn current source on; heat wire by slowly increasing current. When wire reaches melting point of tin, which is 232° C, foil will melt and weight will fall. Observe position of scratch at instant that weight falls.
5. Use this temperature in Equation 1 to calculate linear coefficient of expansion of wire; compare it to value in Table 1.
6. List possible sources of error in this experiment. Does position of tin foil along wire make any difference?

SELF CHECK

1. Define linear coefficient of thermal expansion and modulus of elasticity.
2. State the mathematical relationship between a temperature increase and the dimensions of a solid material, considering both length and volume. Define all symbols used.
3. State the mathematical relationship for tensile or compressive forces produced in a confined solid by a temperature increase.
4. State three practical implications of the fact that solids expand and contract as temperatures increase or decrease.
5. Suppose that in the summer, when the temperature is 25° C, pure aluminum wire is stretched tight between two telephone poles, which you may assume do not move. How far must the temperature drop before the wire snaps? Use the numbers in Table 1 and use the value of 4.7×10^8 dyne/cm^2 as the tensile strength of aluminum (i.e., force per unit area at which aluminum will break).
6. Suppose a bridge 2 miles long is constructed of steel girders. How much total expansion should be allowed along the length of the bridge if the expected annual temperature variation covers a range of 40° C? (Even though the low-alloy structural steels used for bridges have different compositions and somewhat different physical properties for

simplicity, use the value of α for stainless steel as given in Table I.)

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